Crockett Johnson: Painter of Theorems

Katie Ambruso Acker
Robert McGee

Acknowledgments

- The Crockett Johnson Homepage
  - http://www.ksu.edu/english/nelp/purple/

- Peggy Kidwell
  - Curator of Mathematics, Smithsonian Institute.
  - We wish to thank the Smithsonian Institute for allowing us to use digital images of Crockett Johnson's paintings.
  - http://americanhistory.si.edu/collections/group_detail.cfm?key=1253&gkey=192

Biography

- **1906 born David Johnson Leisk**
- **Education**
  - 1924 Cooper Union
  - 1925 New York University
- **Cartoons**
  - 1934-1940 *New Masses*
  - 1940-1943 *Colliers, The Little Man with Eyes*
  - 1942-1953 *Barnaby*
- **1940 married Ruth Krauss, author**
- **Children's Books**
  - 1952-1965 *Harold and the Purple Crayon*
- **Mathematics Paintings**
- **1975 Dies of lung cancer**

Crockett Johnson on His Paintings

"In my geometric paintings, I use, as intrinsic tools the mathematical geometry and the mathematical methods I, as a desultory and very late scholar, have been able to absorb."

C. Johnson, On the Mathematics of Geometry in my Abstract Paintings, *Leonardo* ***5**, 1972.*

Nine-Point Circle

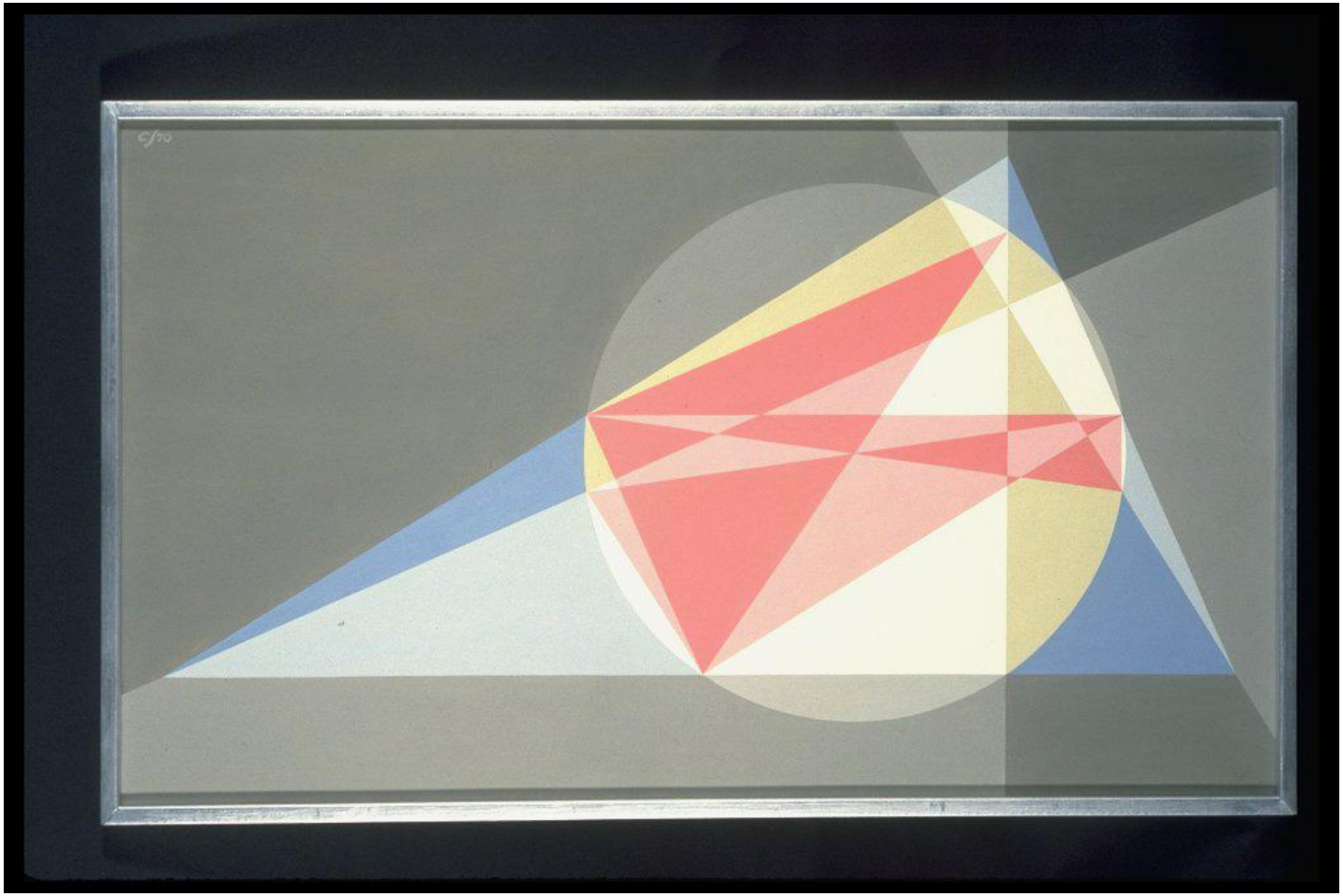
3 points located at the midpoints of the sides of a triangle.

3 points from the feet of the altitudes from each side.

3 points located at the midpoints between the orthocenter and the vertices of the triangle.

Orthocenter-the point of concurrency of the altitudes of a triangle.

Nine-Point Circle



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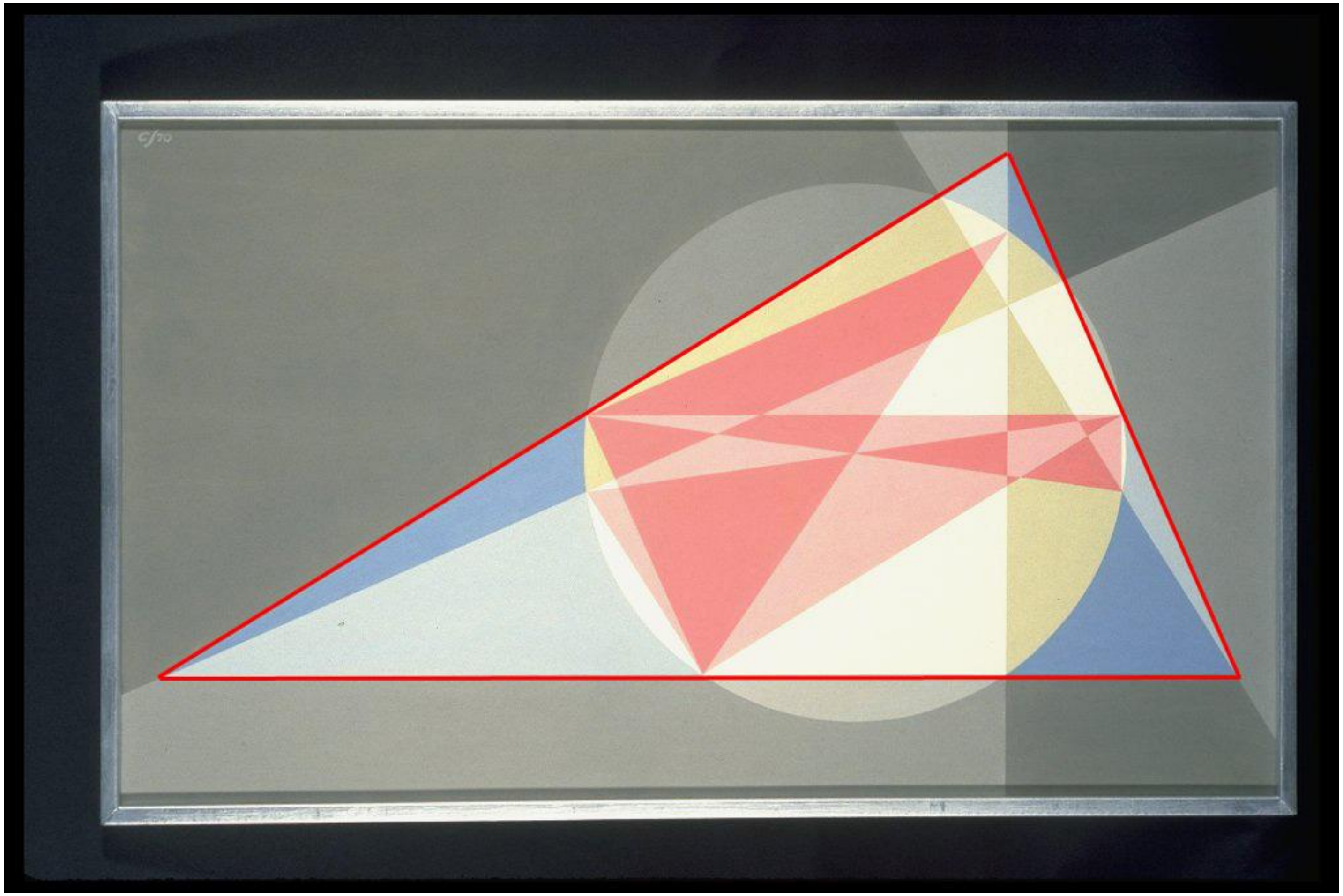

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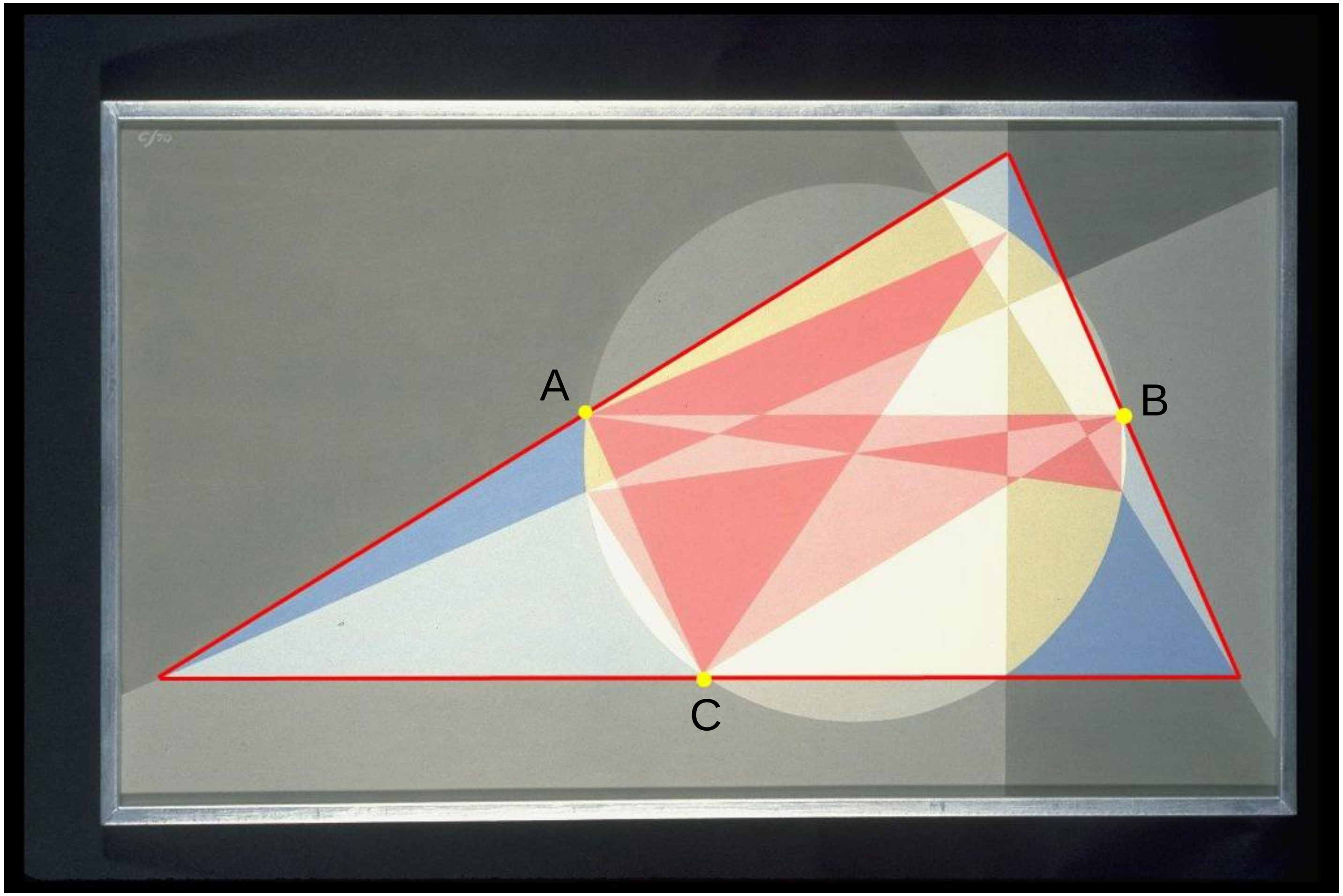
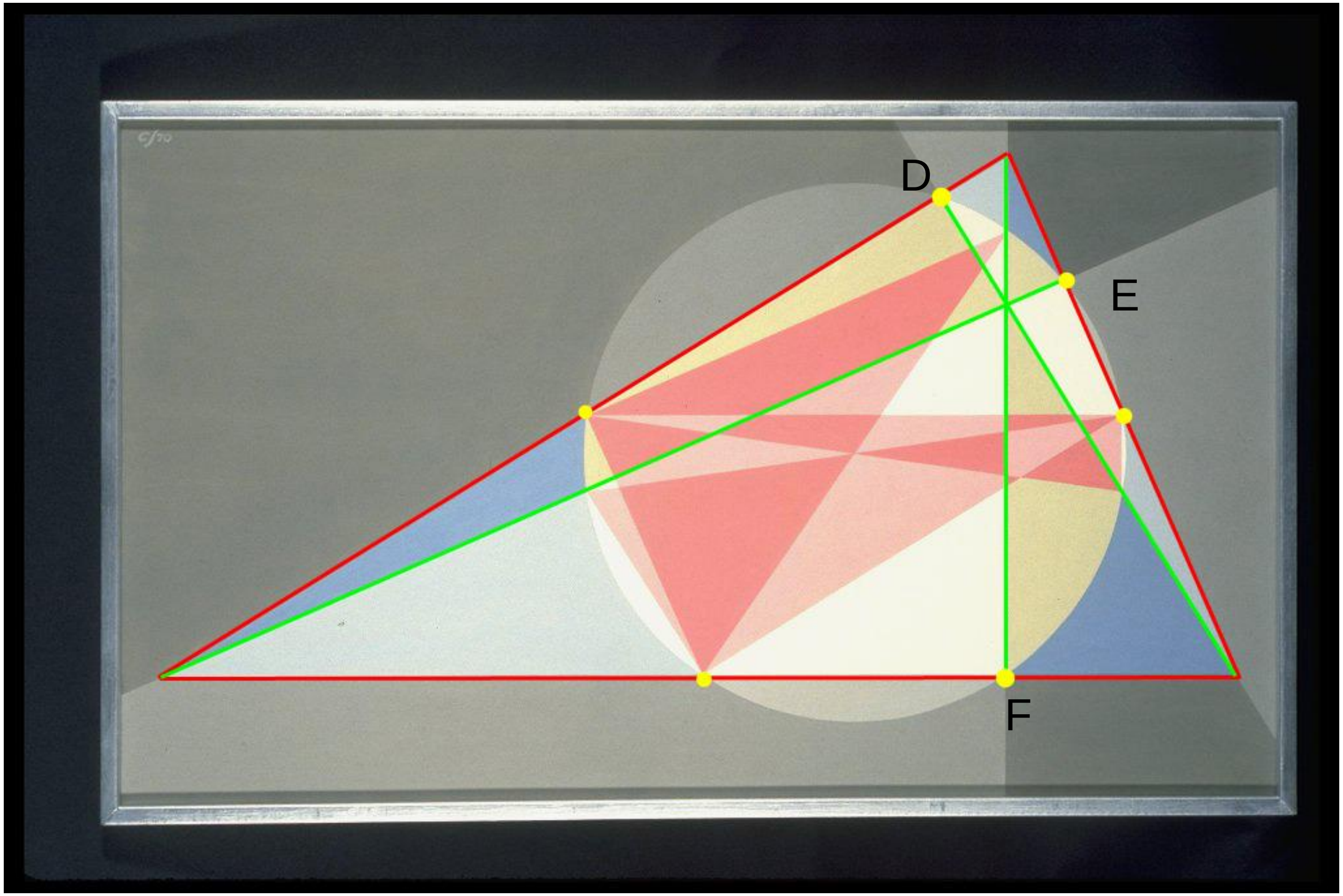


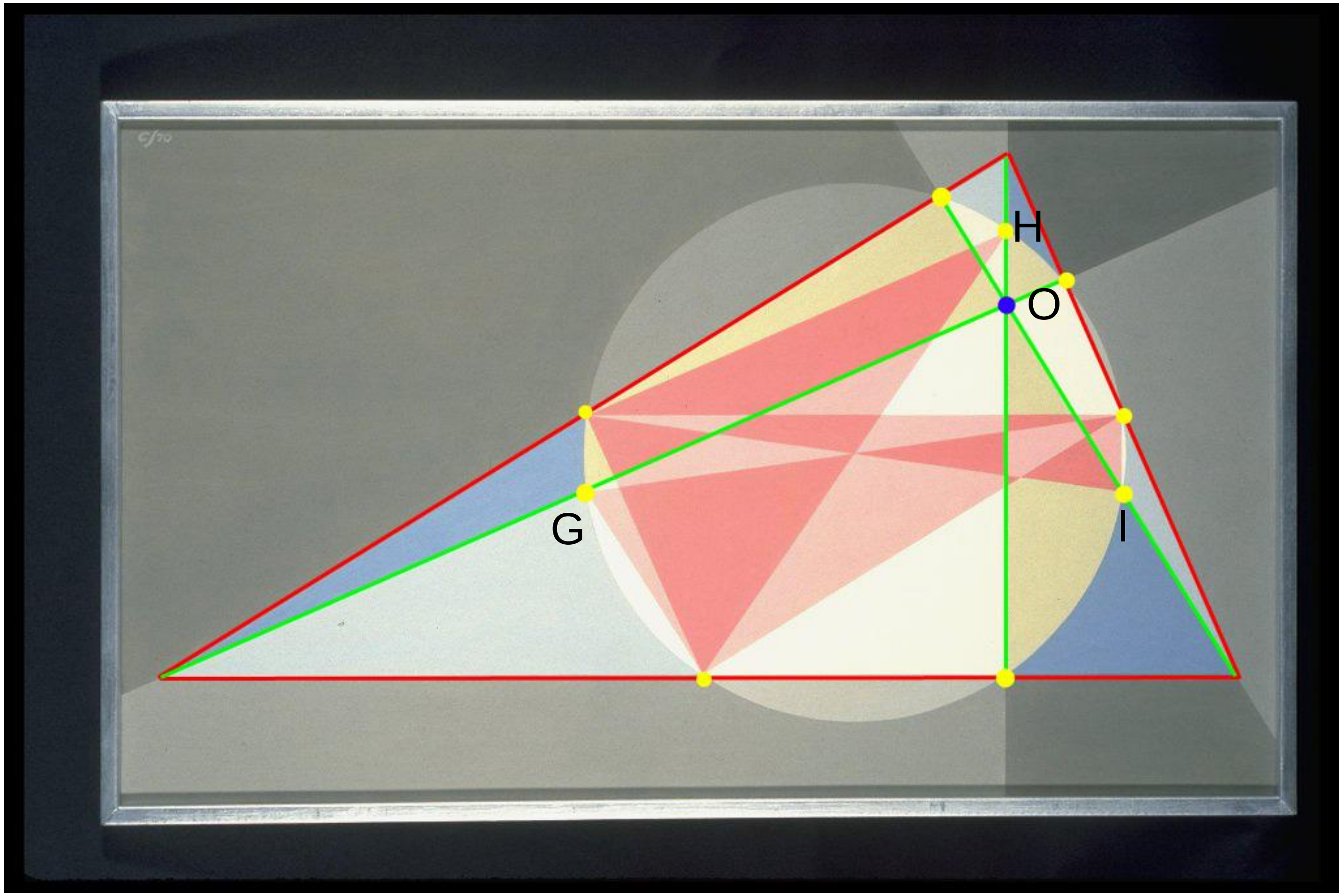
A, B, and C are the midpoints of the sides of the triangle.

Nine-Point Circle



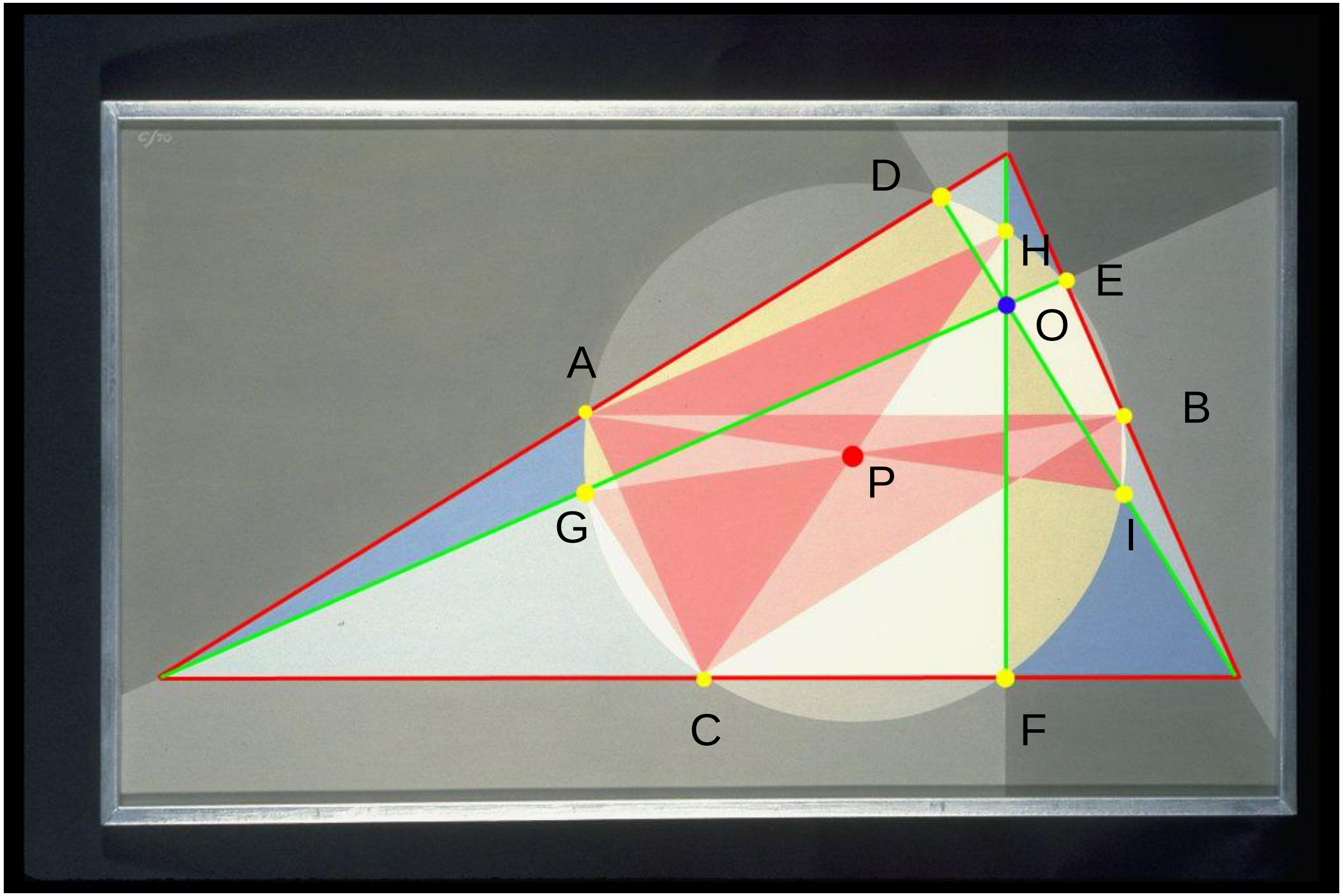
D,E, F are the feet of the altitudes.

Nine-Point Circle

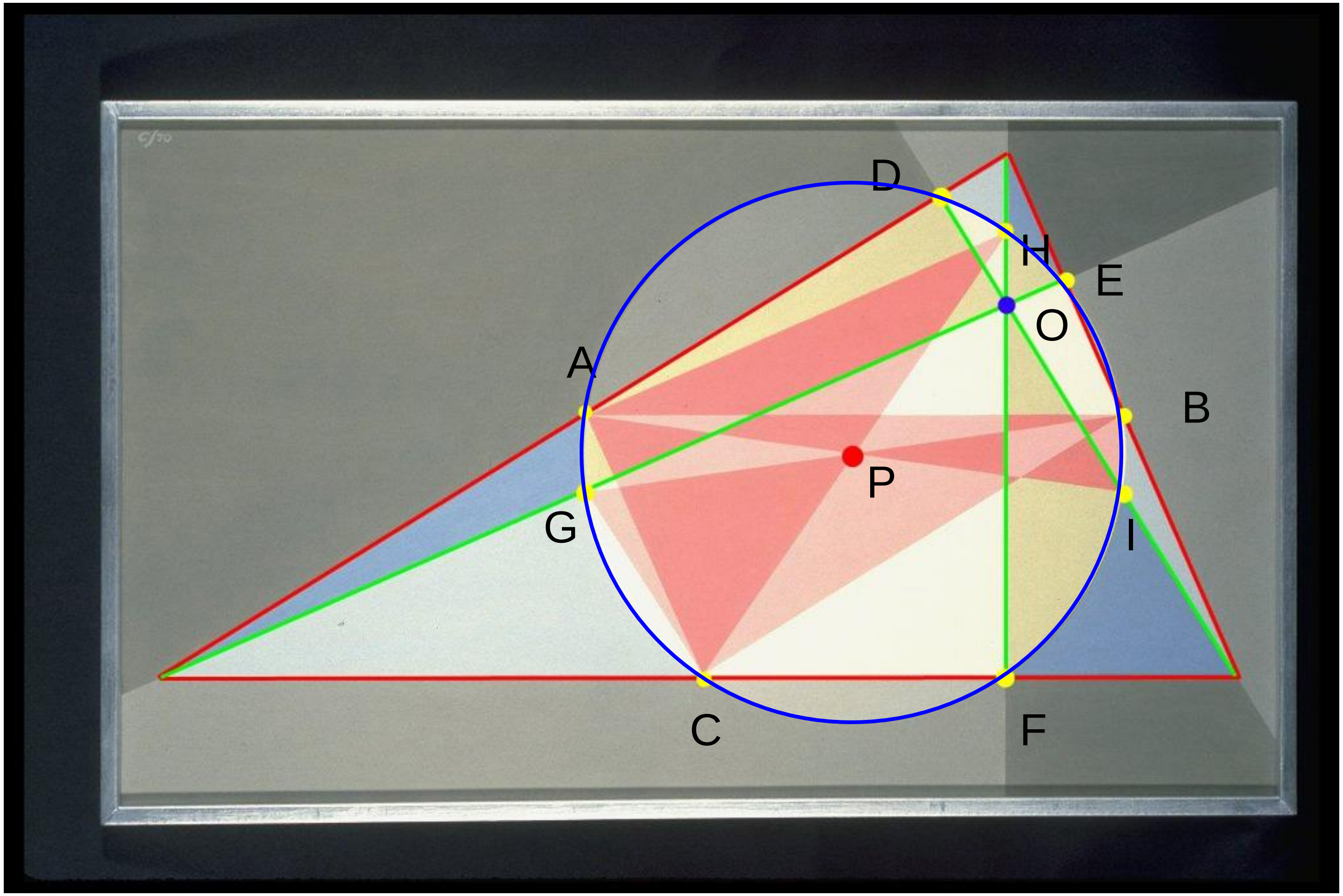


G, H, I are the midpoints between O, the orthocenter, and the vertices of the triangle. These points are also known as Euler Points.

Nine-Point Circle



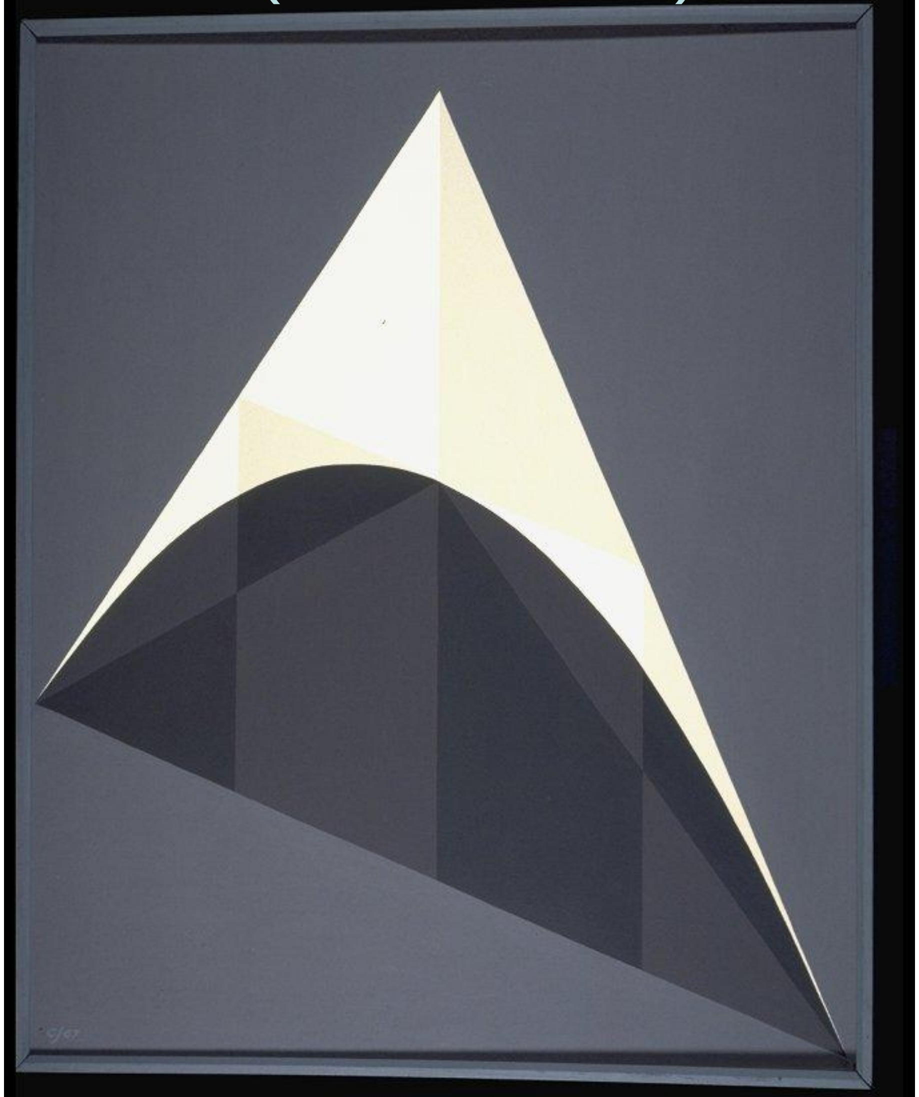
Nine-Point Circle



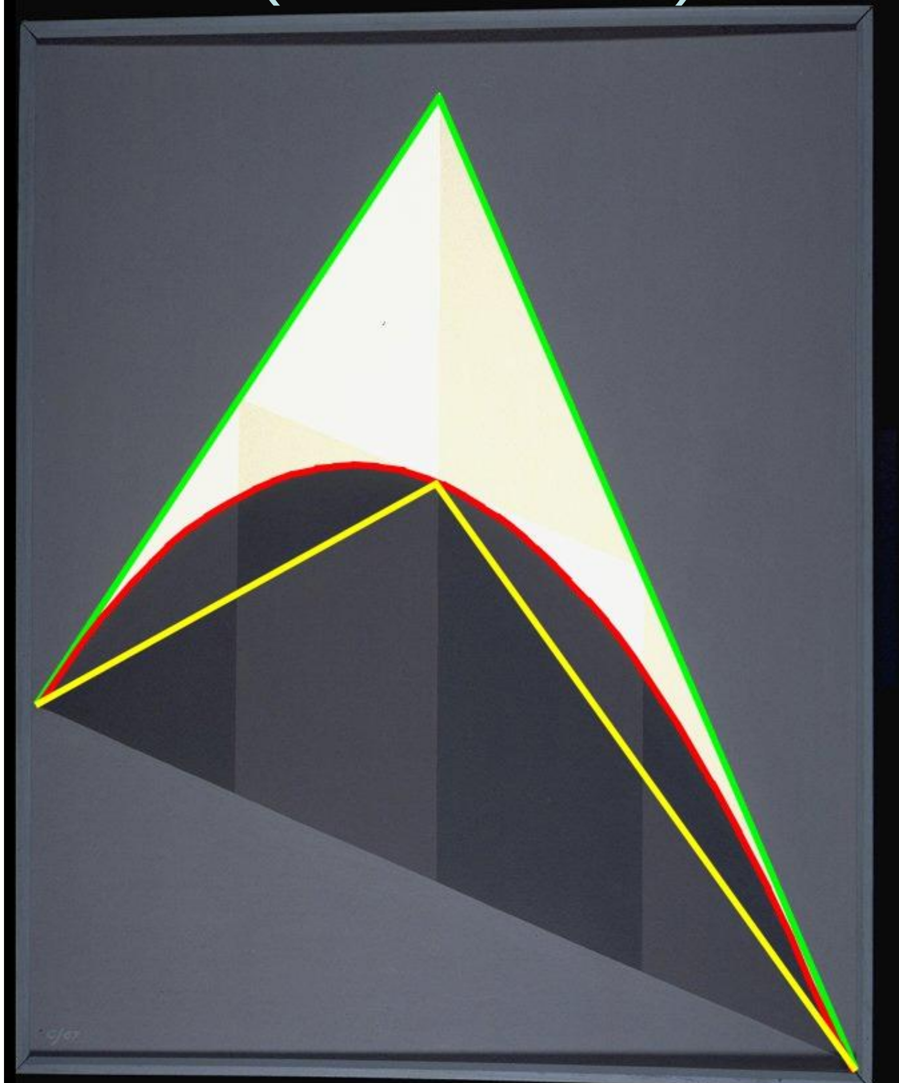
Parabolic Triangles (Archimedes)

The theorems that Archimedes proved were that the area of the parabolic section was equal to 2/3 the area of the parabolic triangle and 4/3 the area of the inscribed triangle. The parabolic triangle is obtained by drawing the tangents to the parabola at the endpoints of the base of the parabolic section.

Parabolic Triangles (Archimedes)



Parabolic Triangles (Archimedes)



Parabolic area is 2/3 of the area of parabolic triangle 4/3 of the area of the incribed triangle

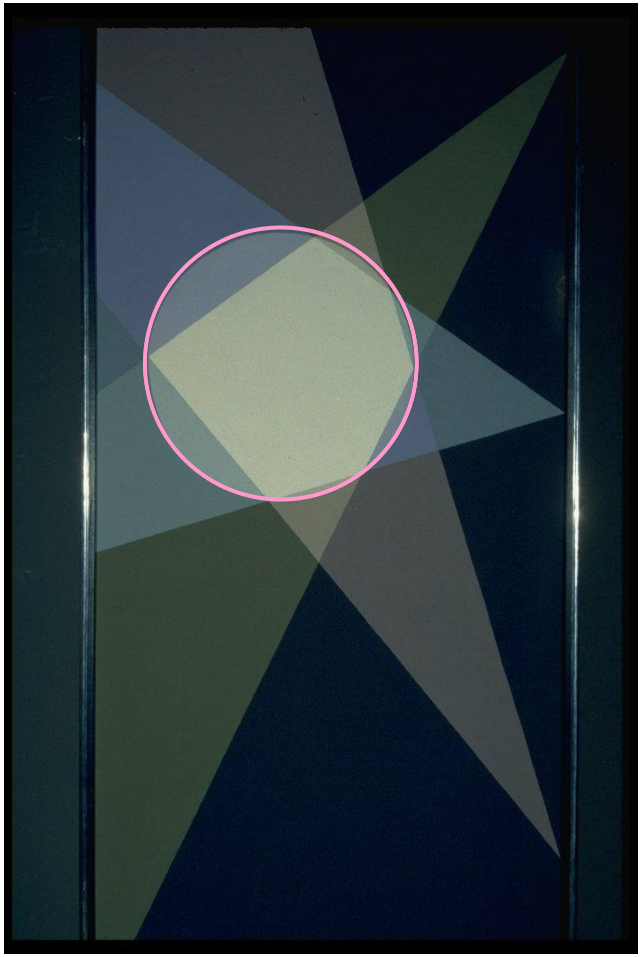
“Mystic” Hexagon

Pascal’s Hexagon Theorem

The three points of intersection of the opposite sides of a hexagon inscribed in a conic section lie on a straight line.

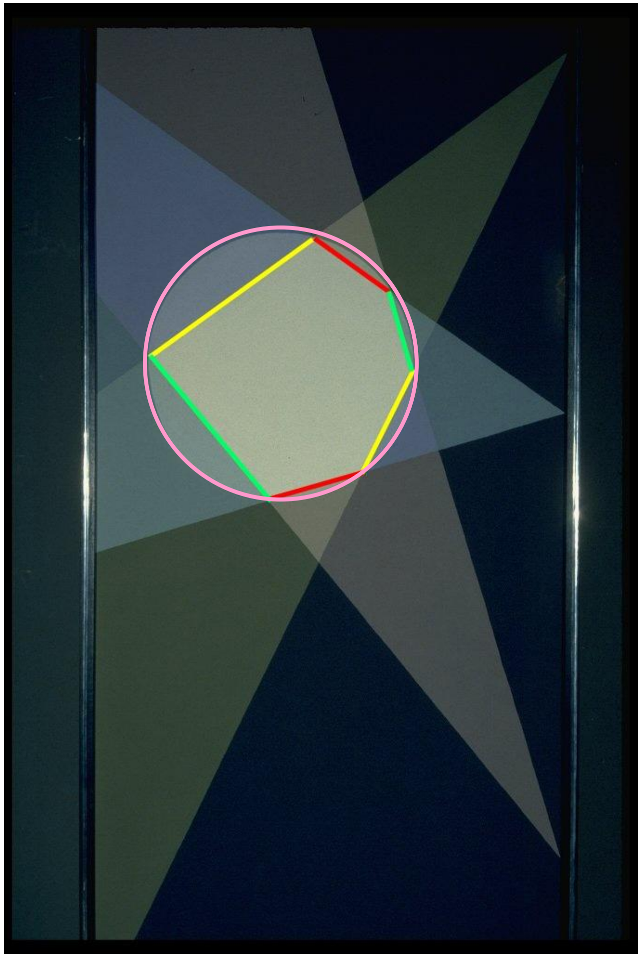
“Mystic” Hexagon

“Mystic” Hexagon



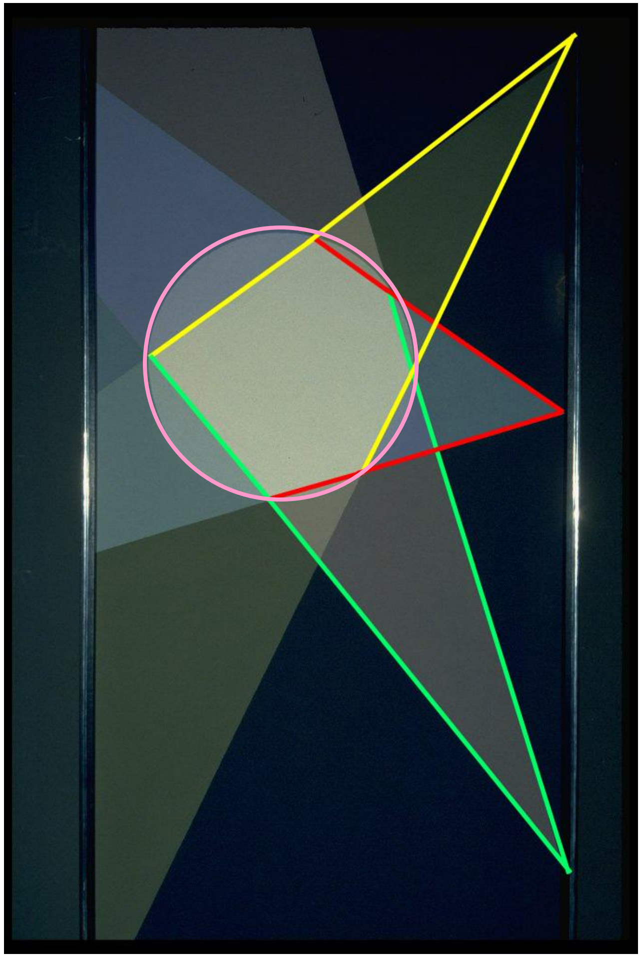
Begin with a circle

“Mystic” Hexagon



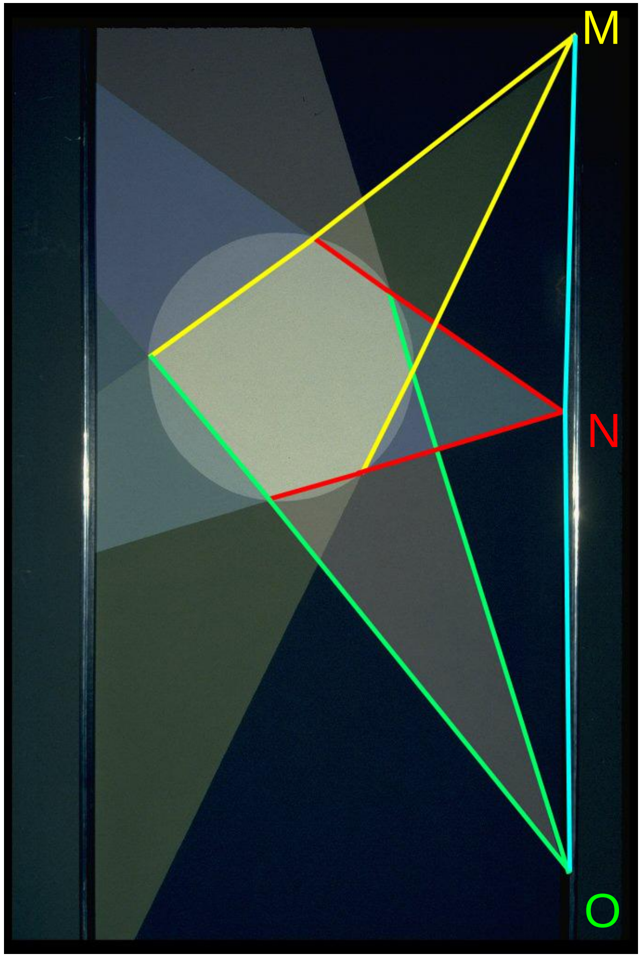
Inscribe a hexagon

“Mystic” Hexagon



Extend the sides.

“Mystic” Hexagon



M,N,O are collinear.

Pythagorean Theorem

Proposition 47 Euclid Book I

- In a right triangle, the square on the hypotenuse is equal in area to the sum of the squares on the sides.

- There are over 300 proofs of the Pythagorean theorem.

$$a^2 + b^2 = c^2$$

Pythagorean Theorem

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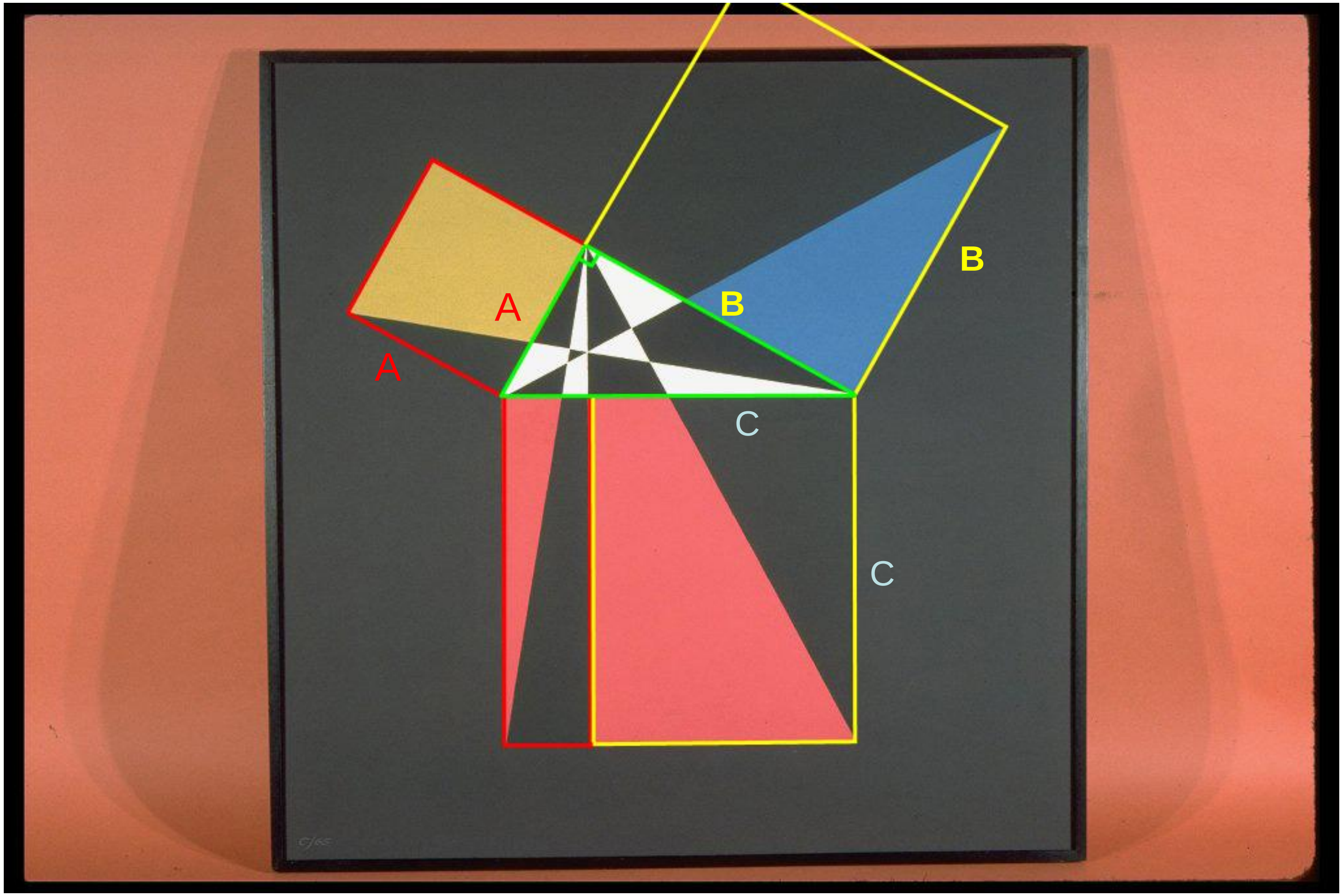
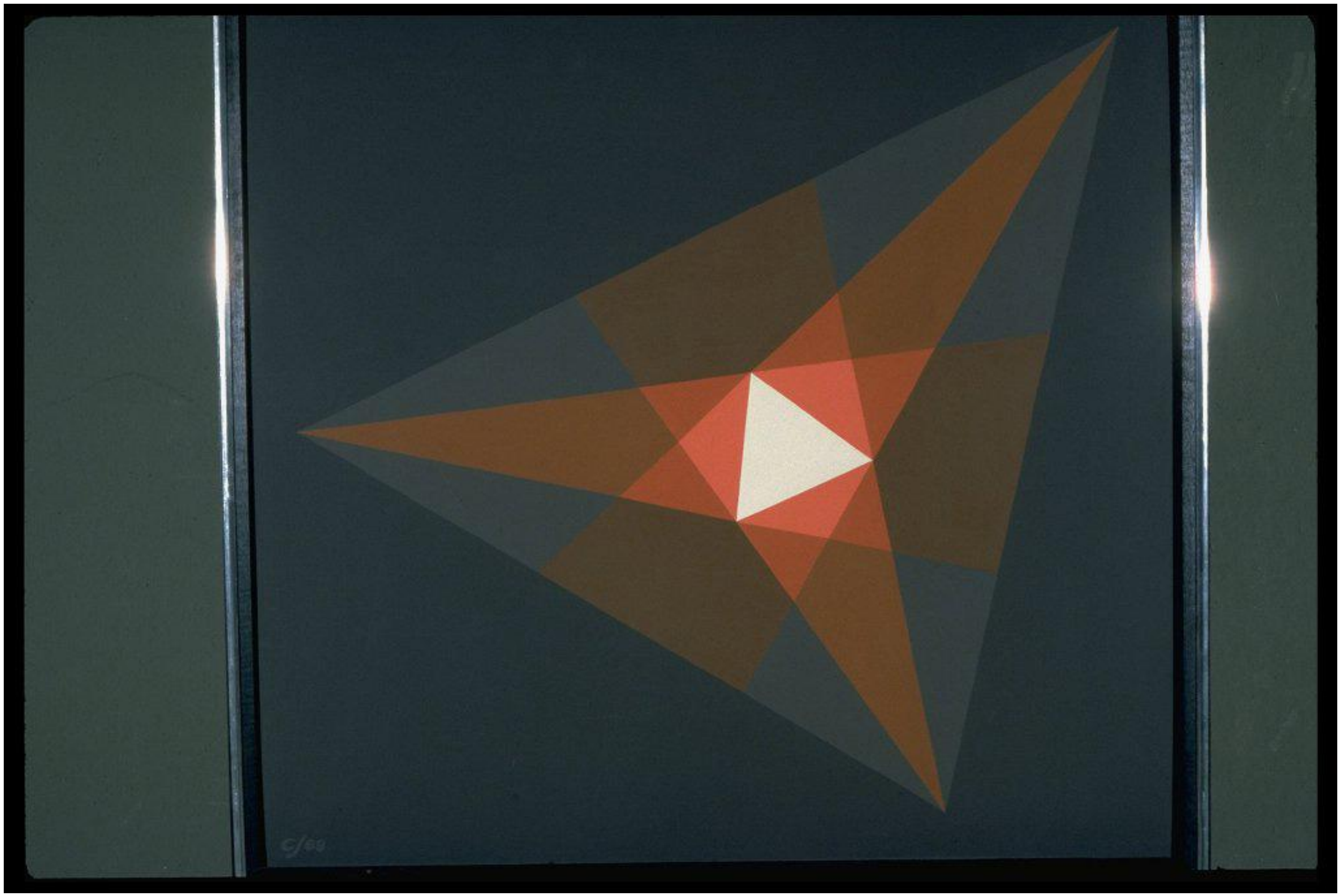
Pythagorean Theorem



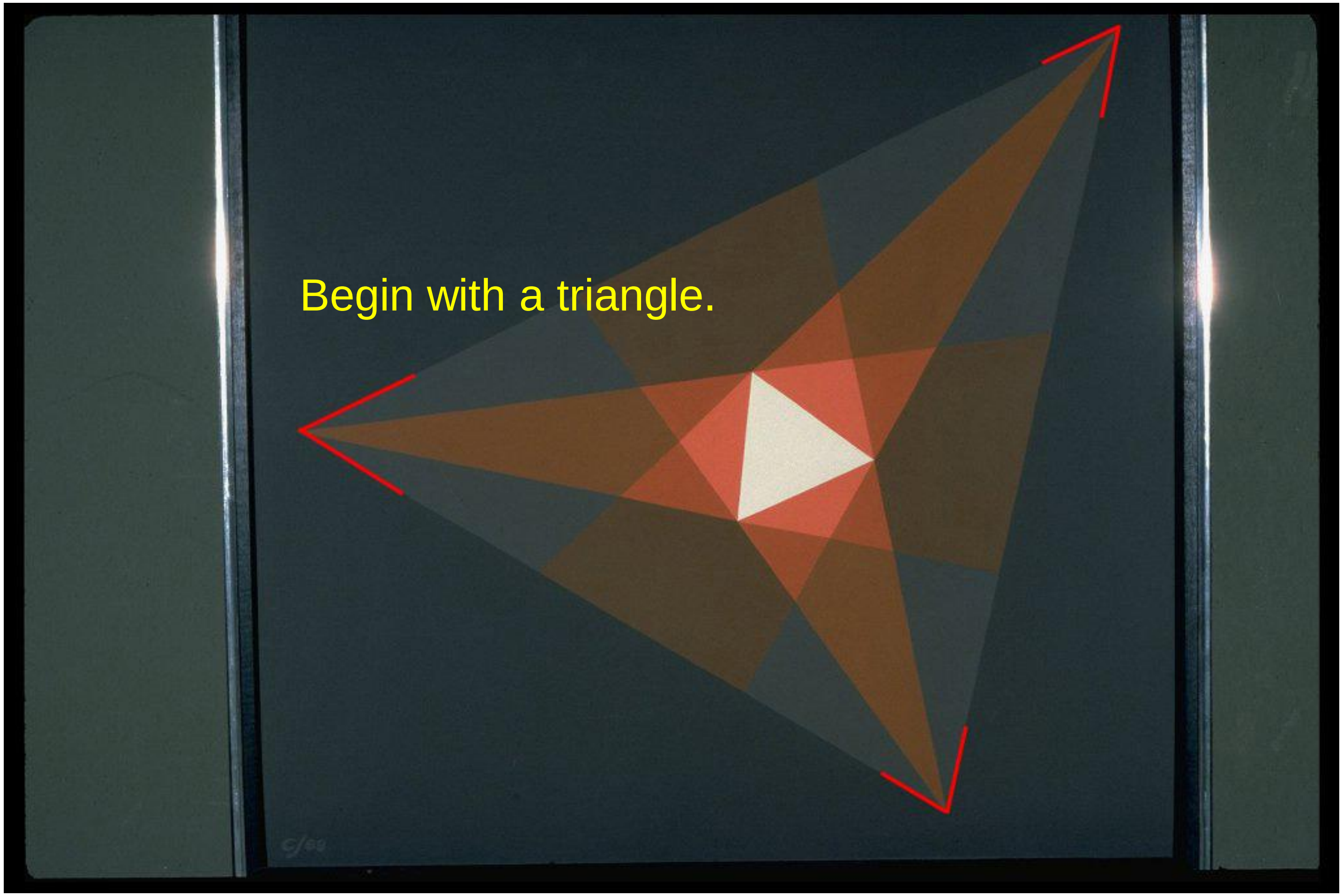
Morley Triangle

Corresponding angle trisectors meet at the vertices of an equilateral triangle.

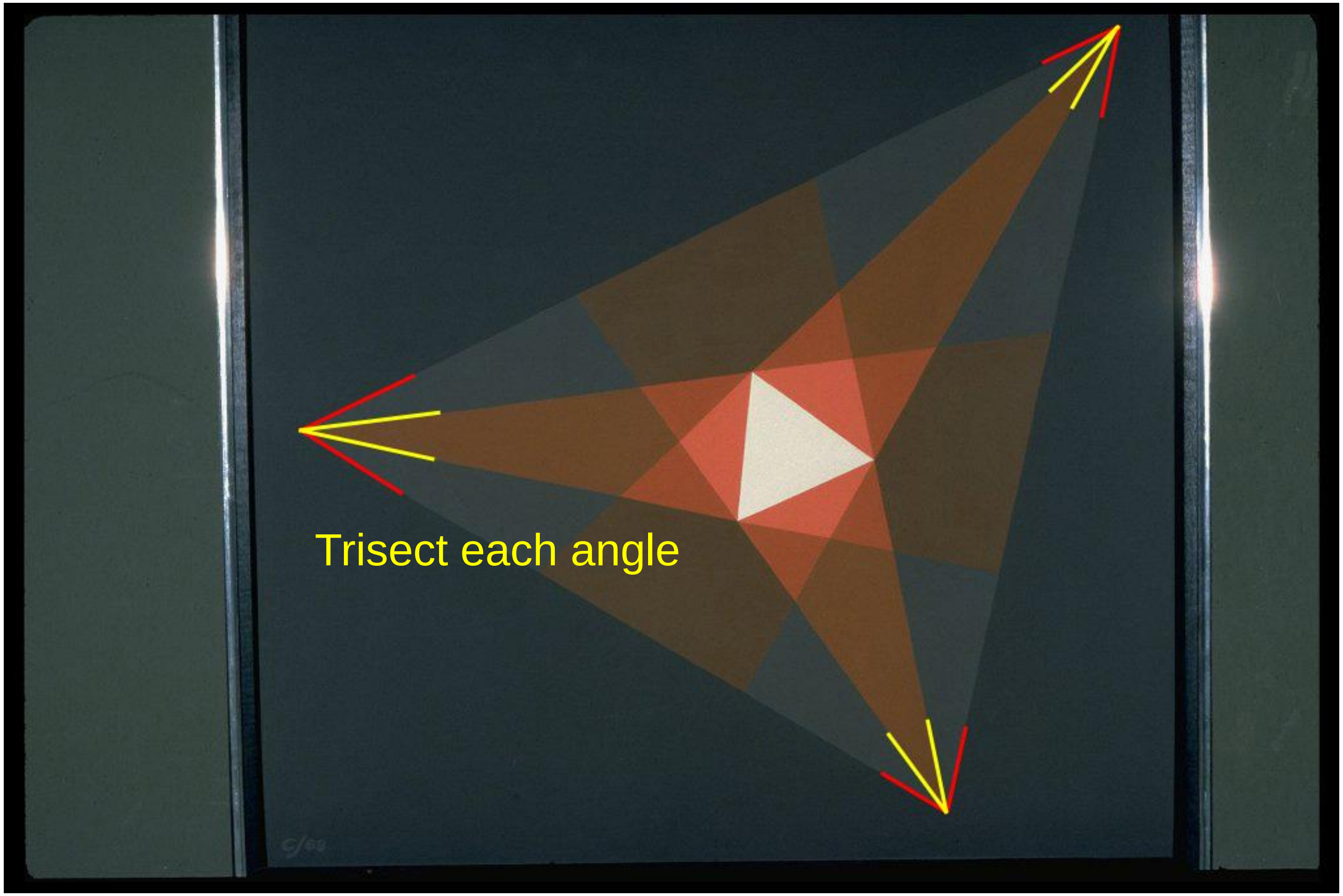
Morley Triangle



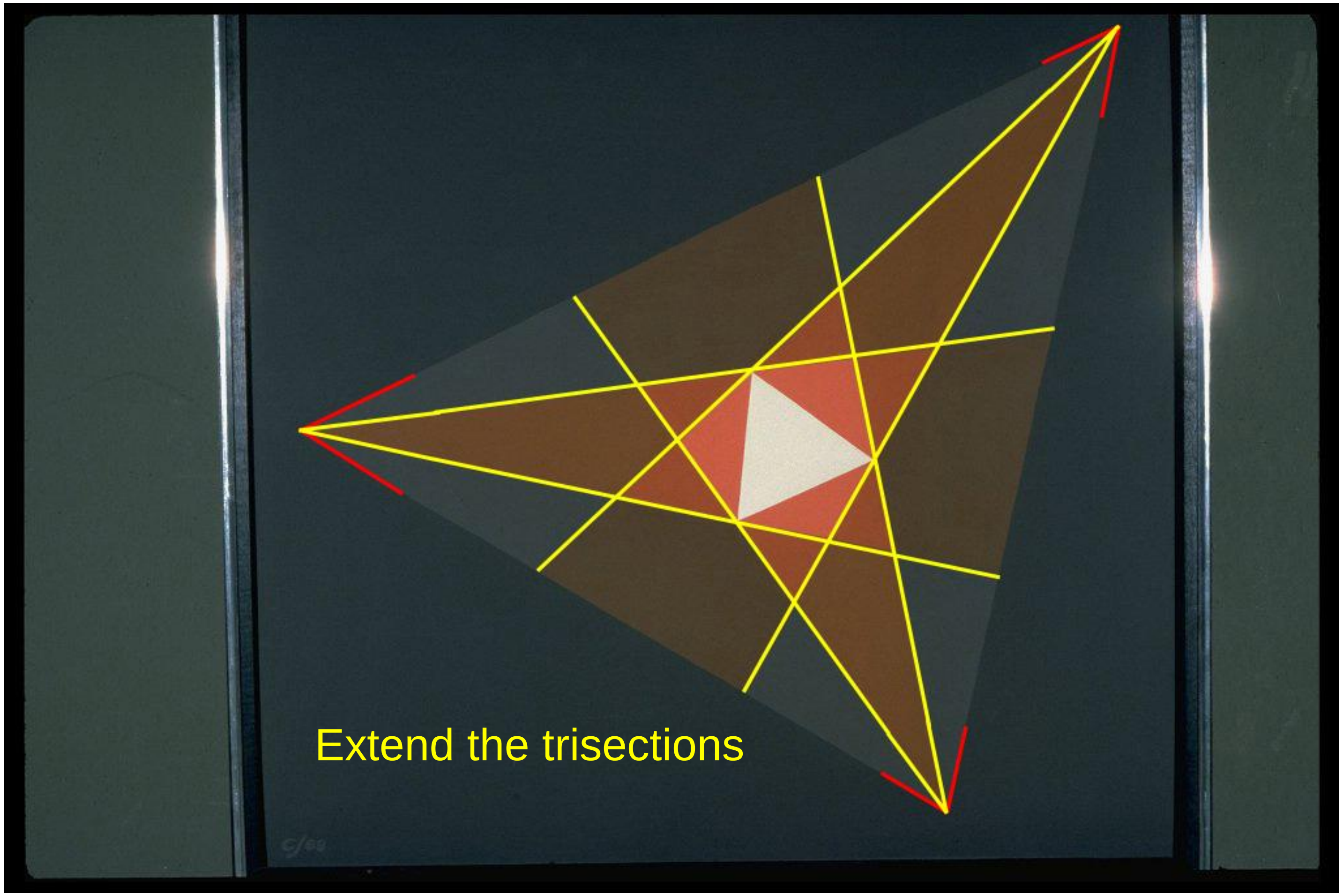
Morley Triangle



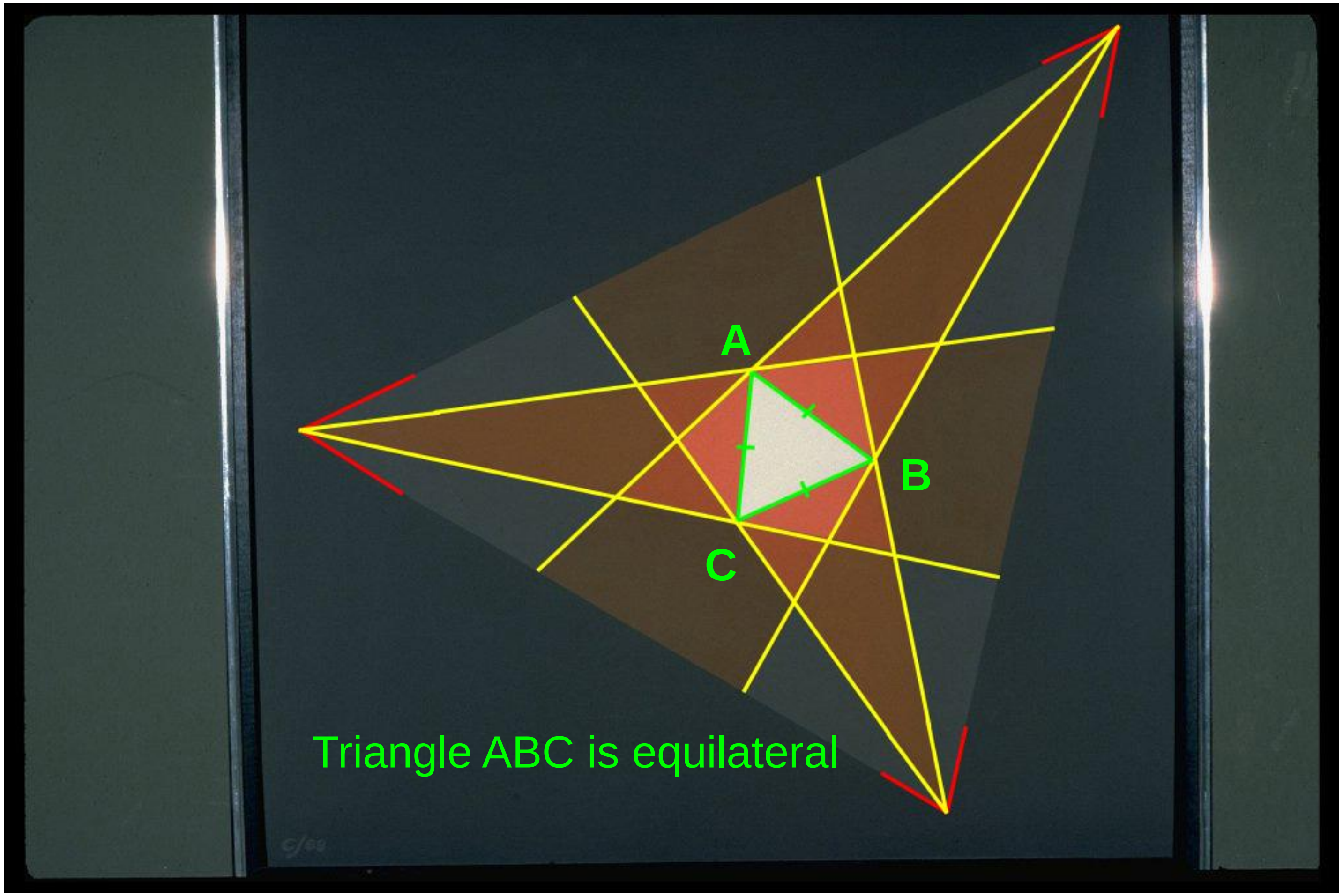
Morley Triangle



Morley Triangle



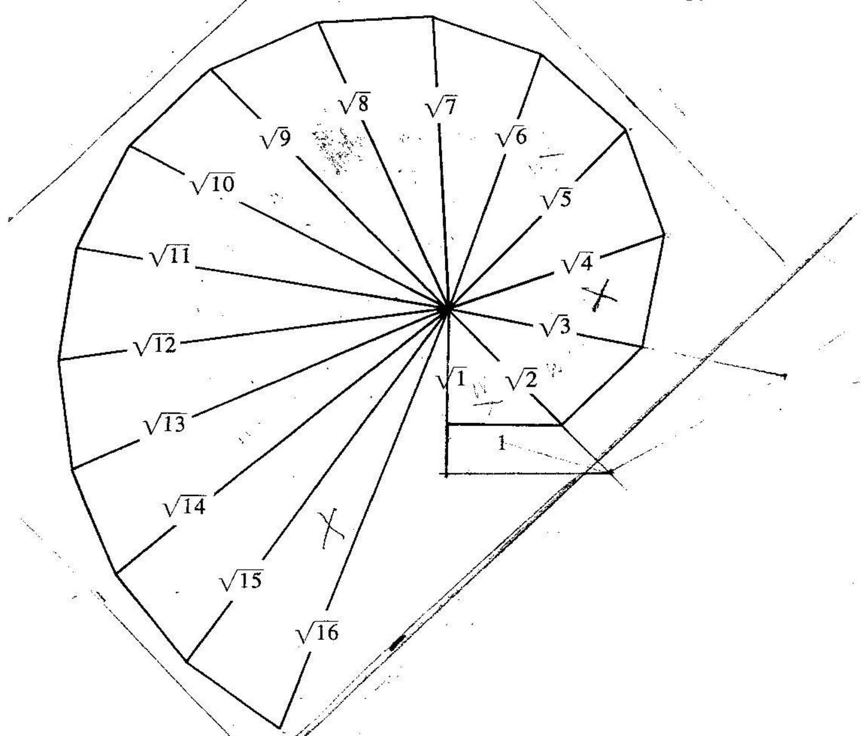
Morley Triangle



Theodorus

- Thought to be a Pythagorean.
- Mathematics tutor to Theaetetus and Plato.
- According to Plato, Theodorus was the first to show that square roots of nonsquare integers from 3 to 17 are incommensurable with 1.

E. Valens, p. 61

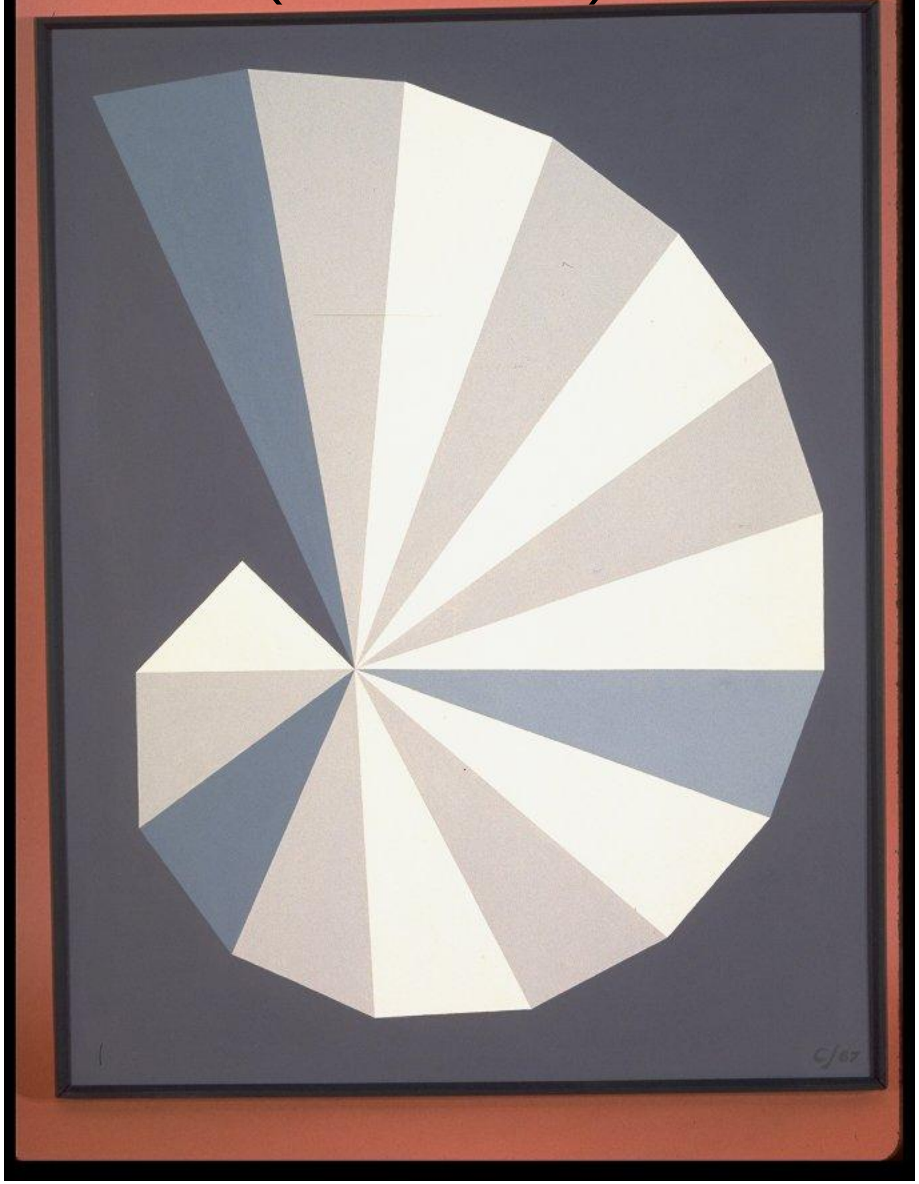


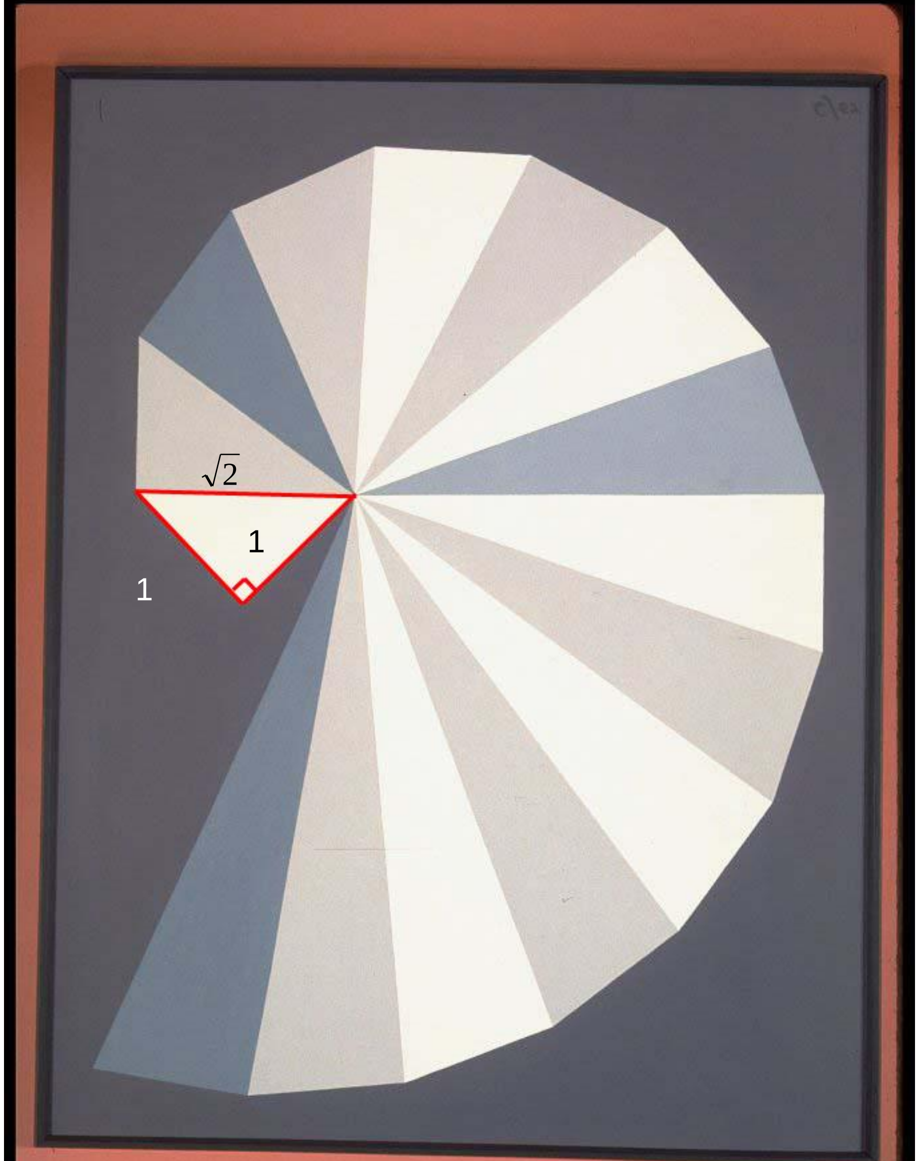
Each new side is derived from the previous side by the Pythagorean theorem. For example, $(\sqrt{14})^2 + 1^2 = (\sqrt{15})^2$.

The first three sides—$\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$—are of particular interest because they are, respectively, the side, the surface diagonal, and the interior diagonal of a unit cube.

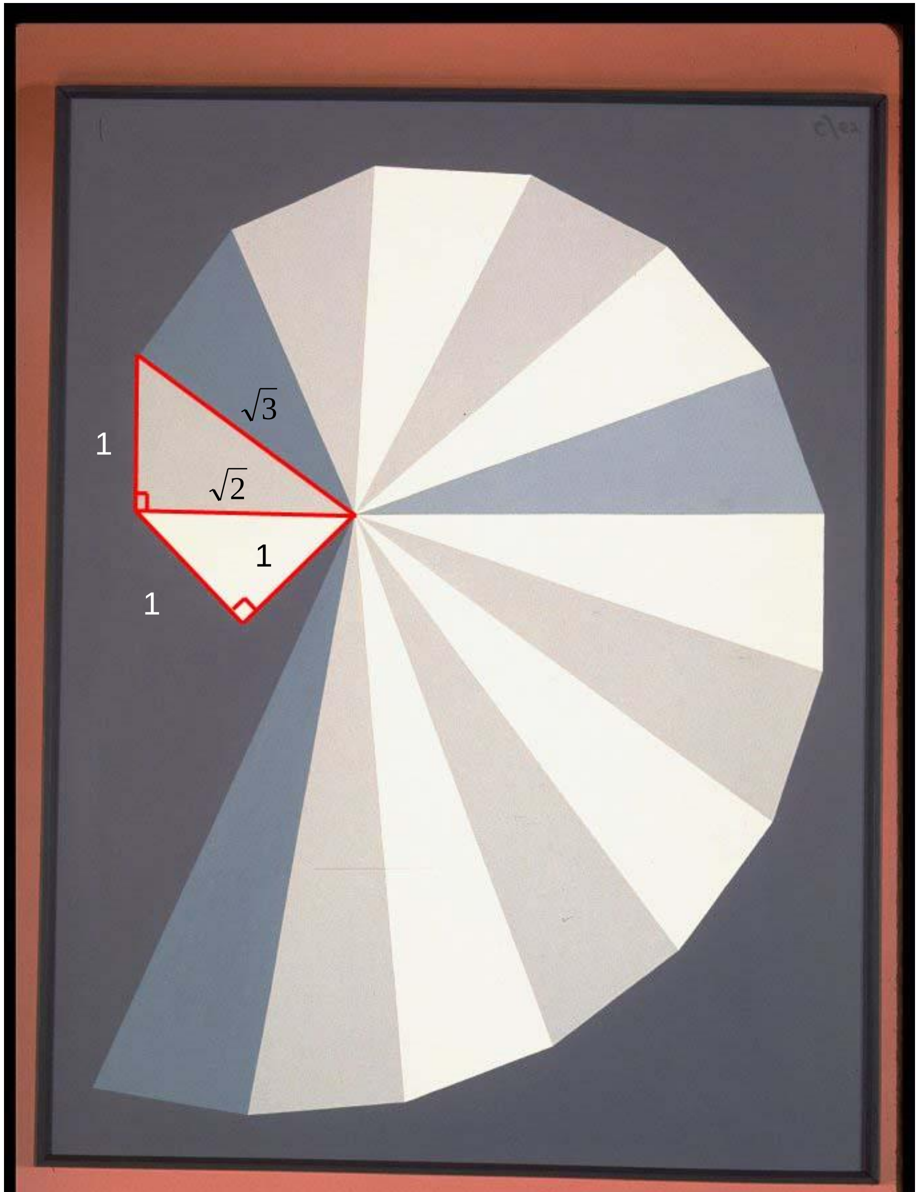
Why did Theodorus stop short of the square root of 17? Plutarch said the Pythagoreans "have a horror for the number 17" because it lies between two somewhat magical numbers: 16, which is a square with an area equal to its perimeter, and 18, which is the double of a square and is also a rectangle (3×6) with an area equal to its perimeter.

Square roots to 16
(Theodorus)





Square roots to 16
(Theodorus)



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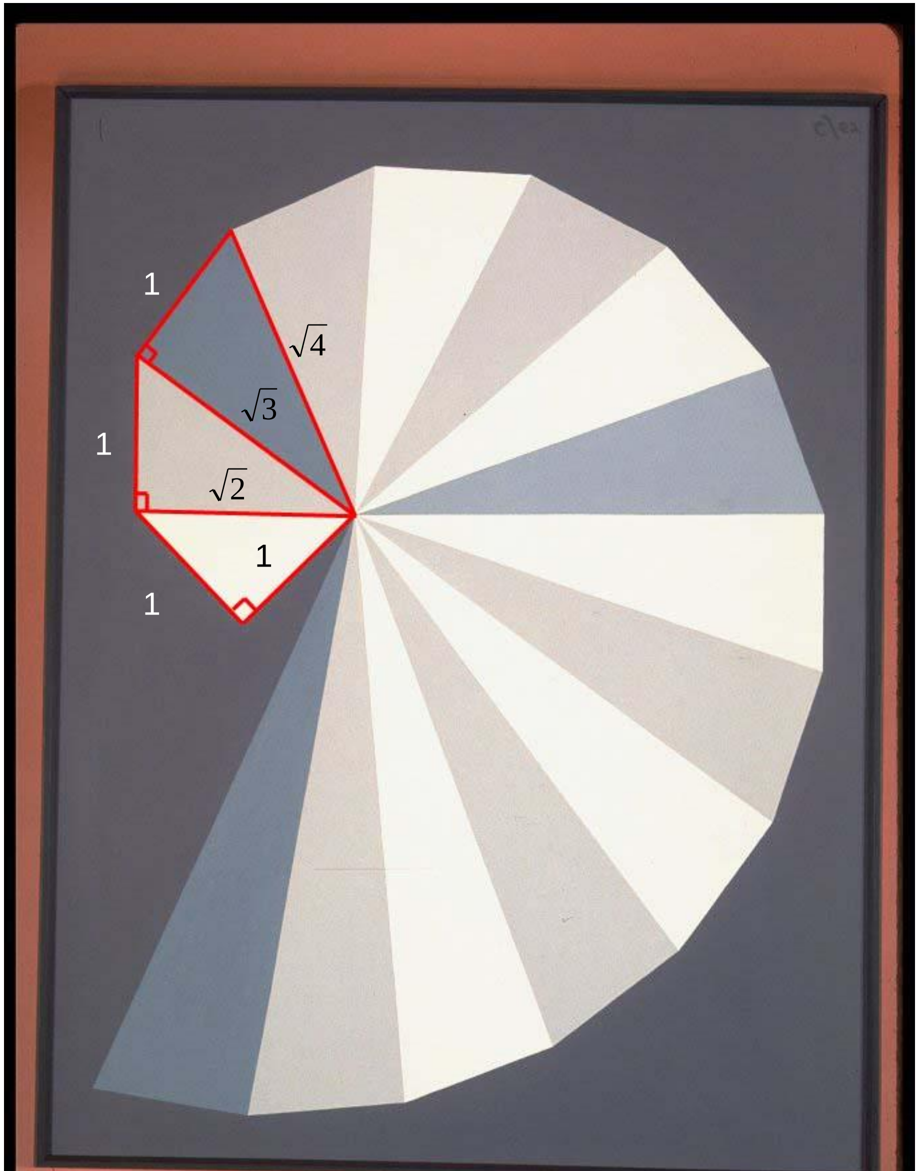

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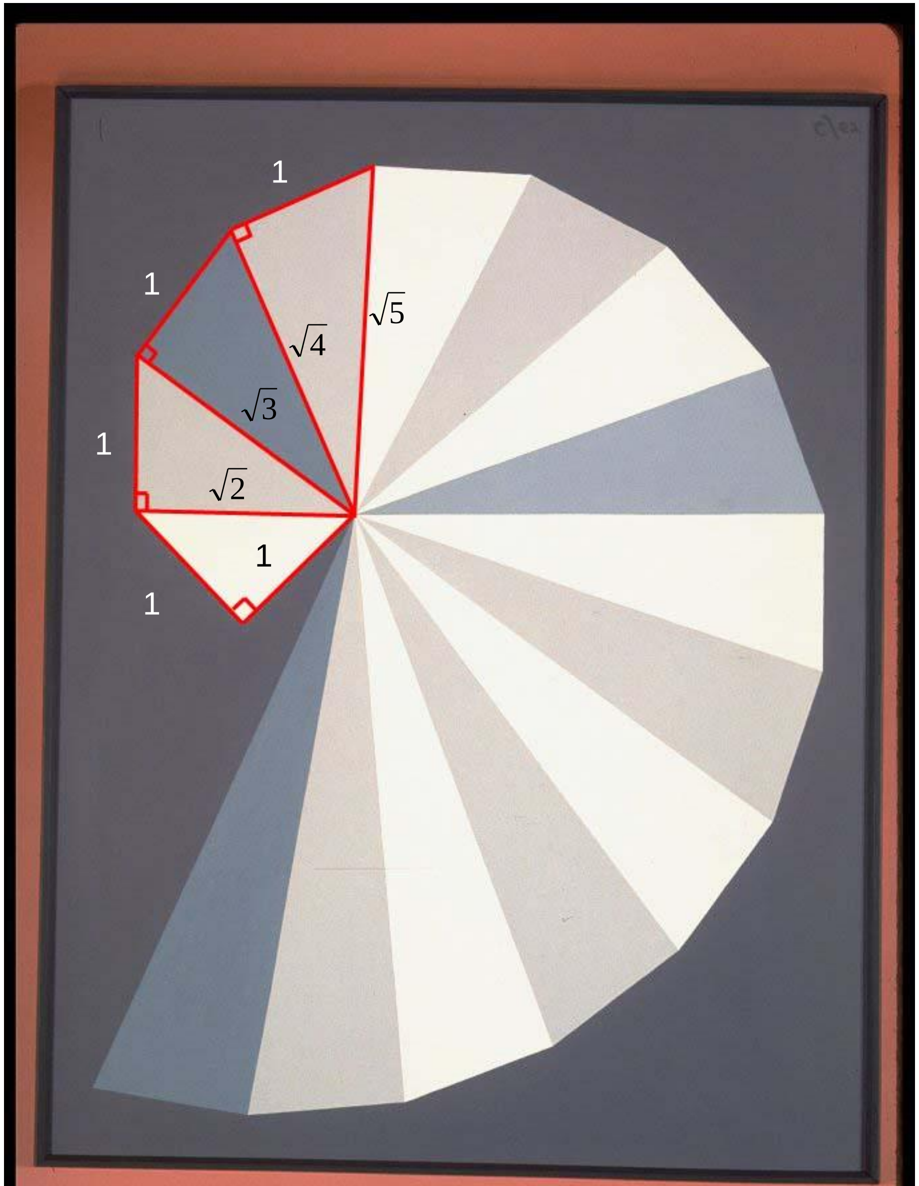

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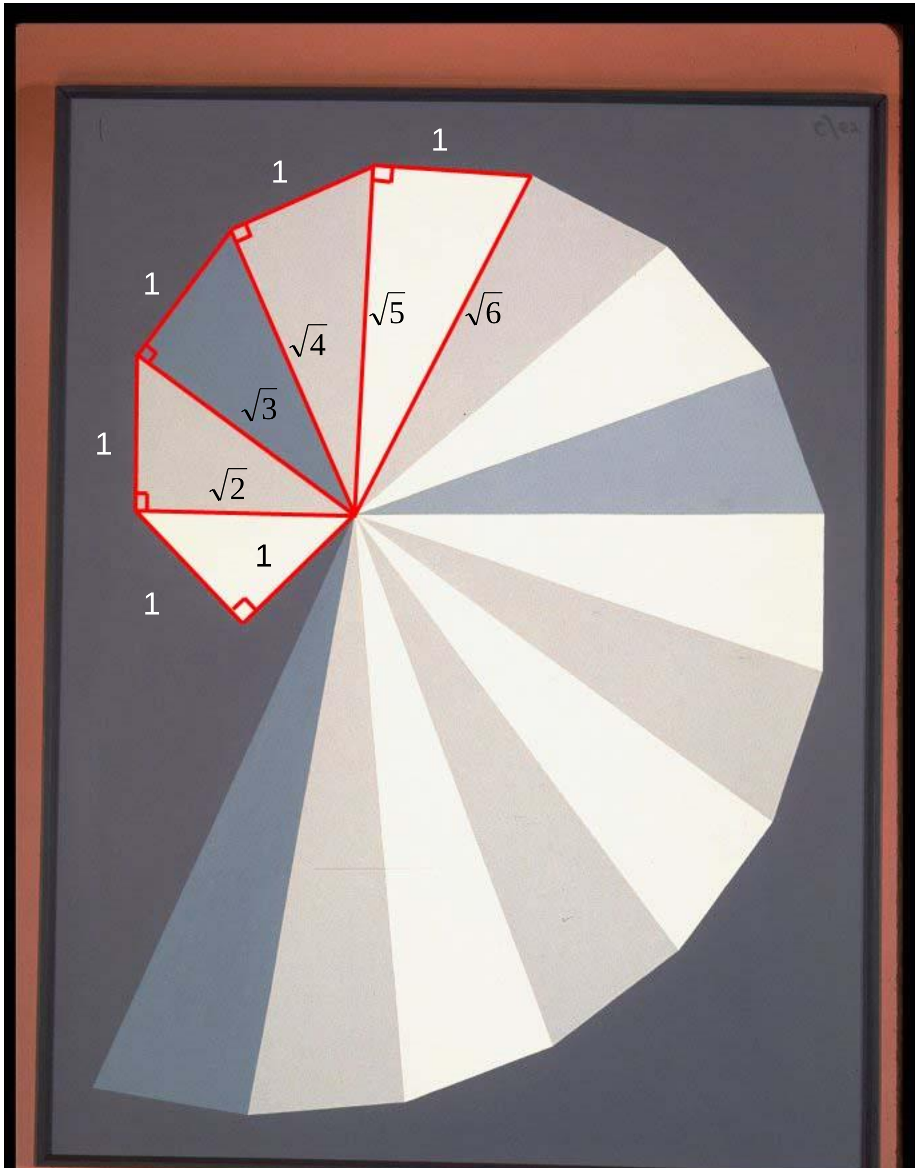

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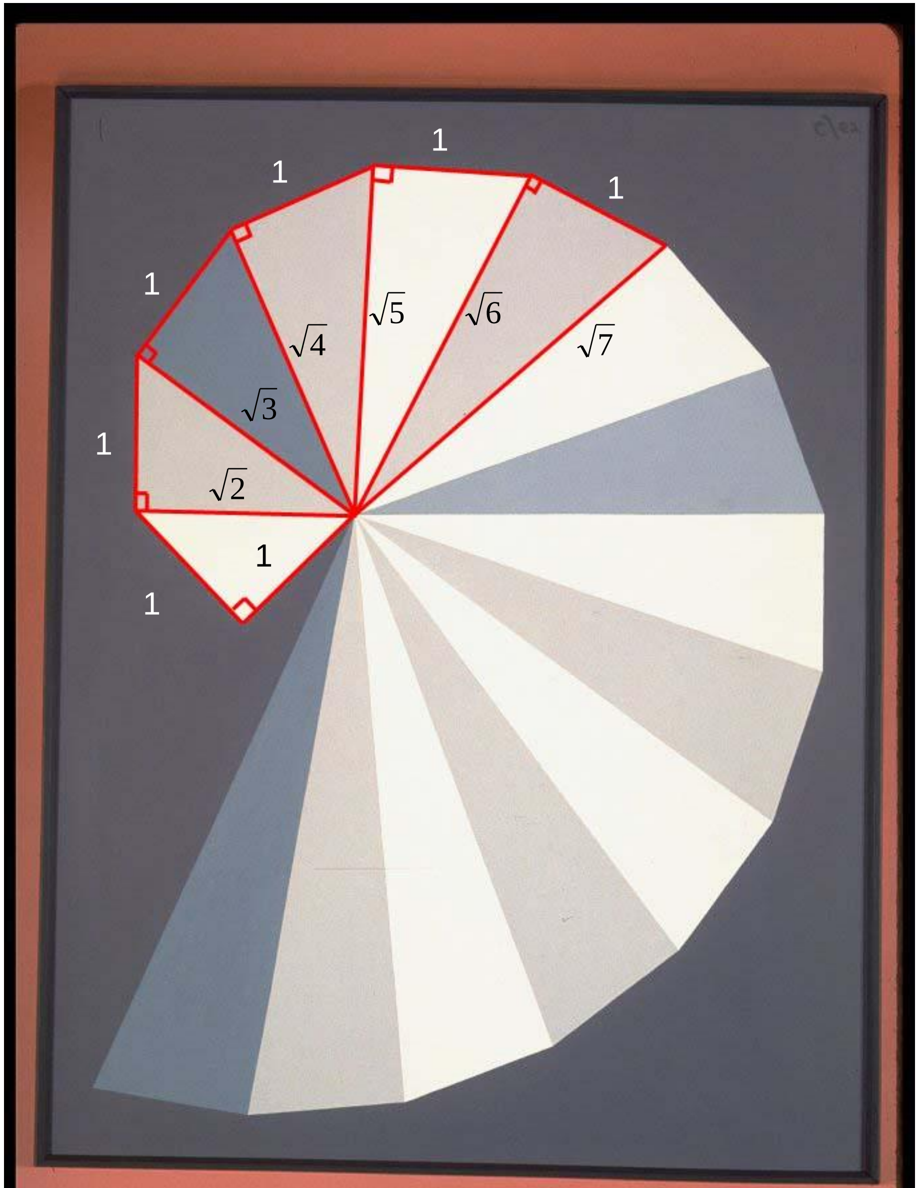

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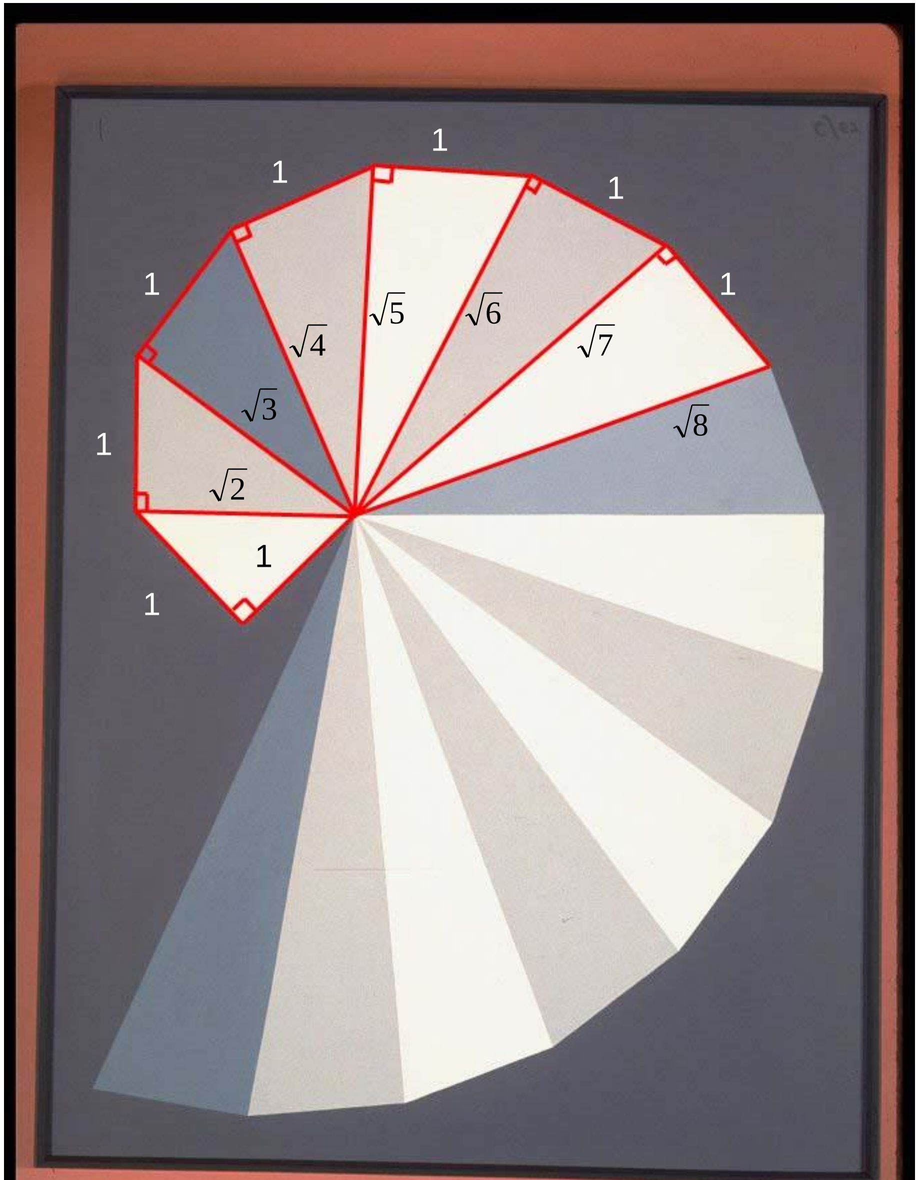

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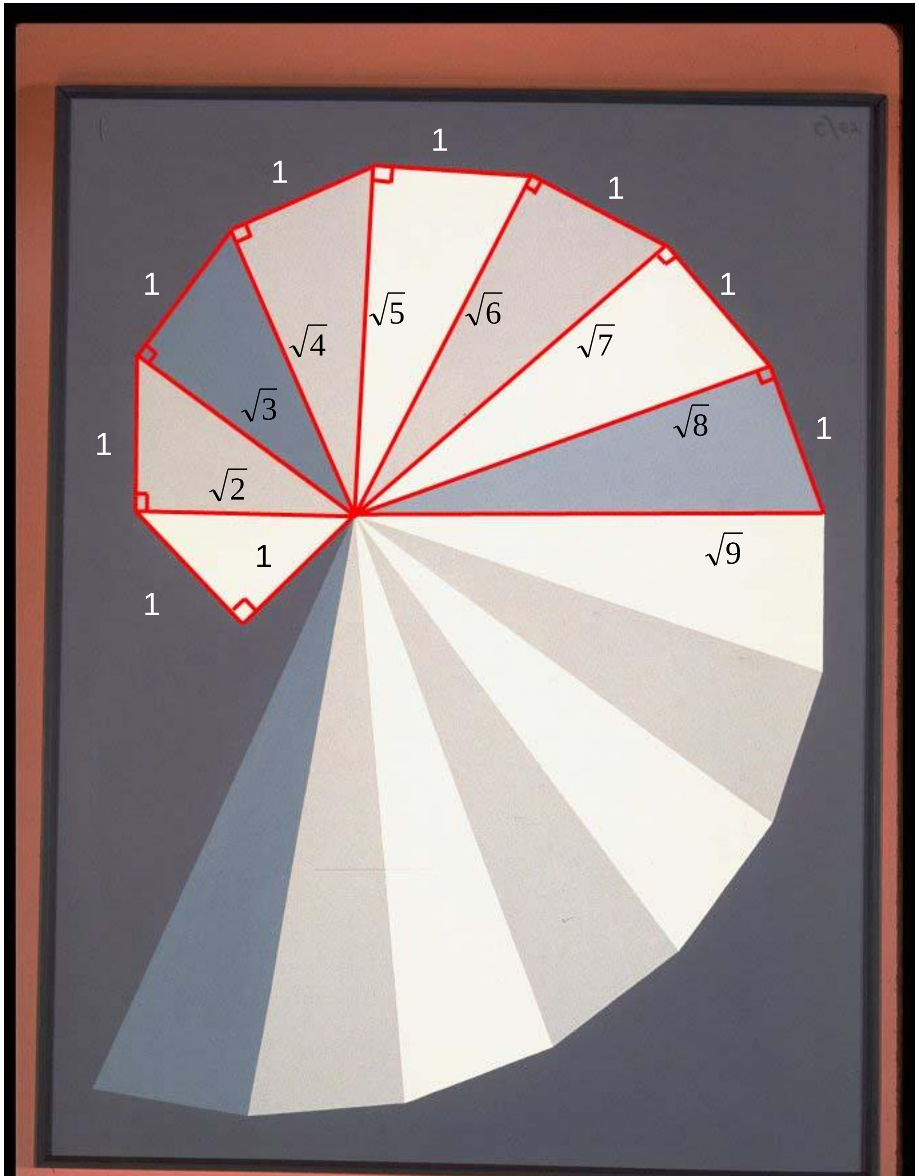

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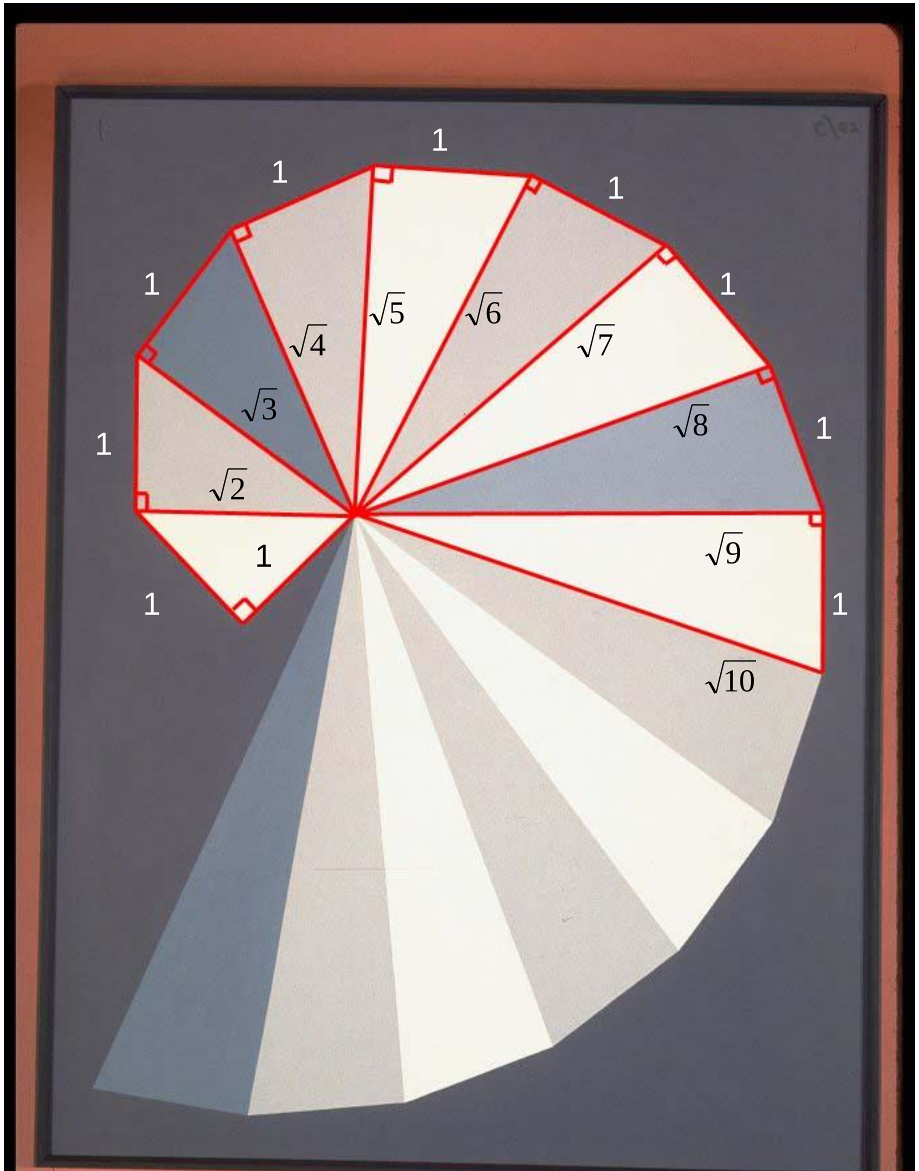

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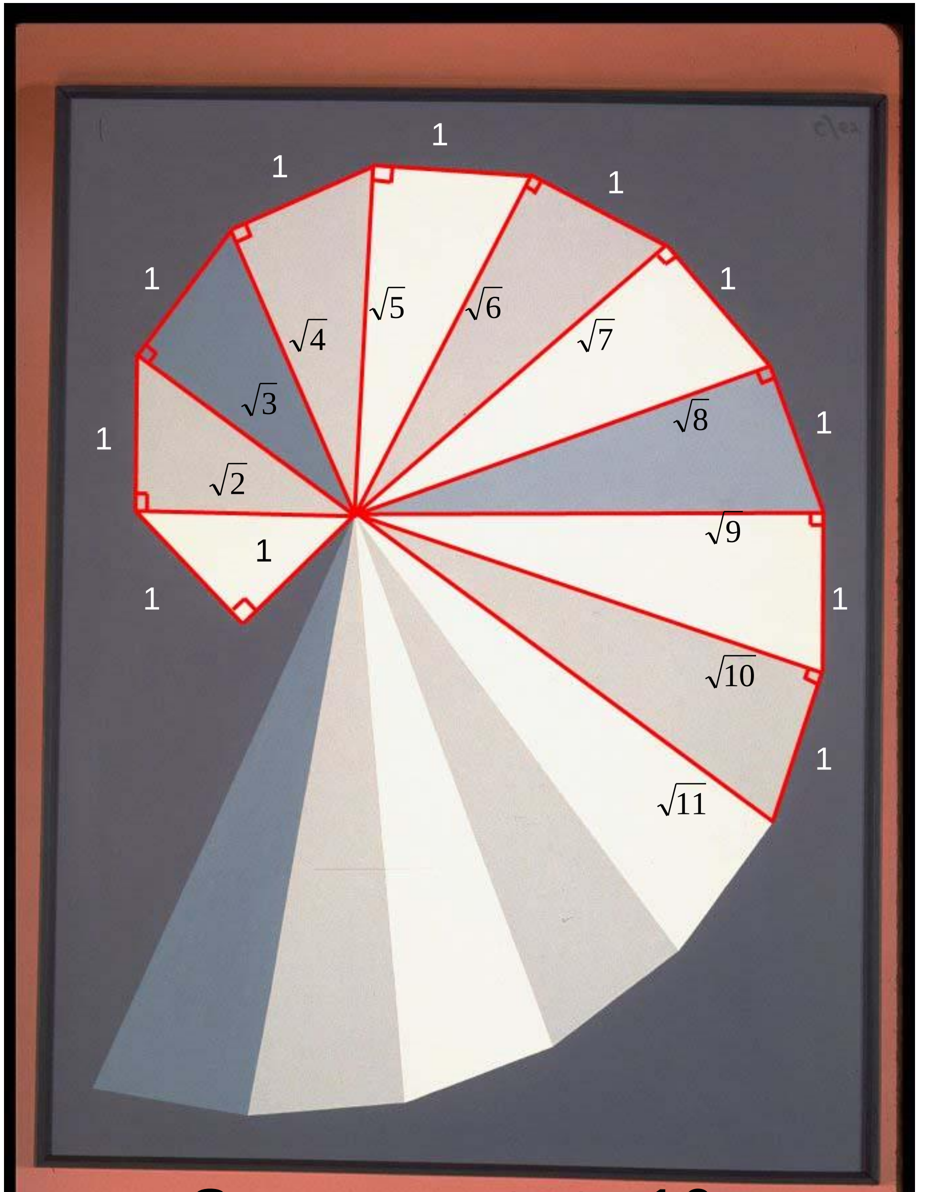

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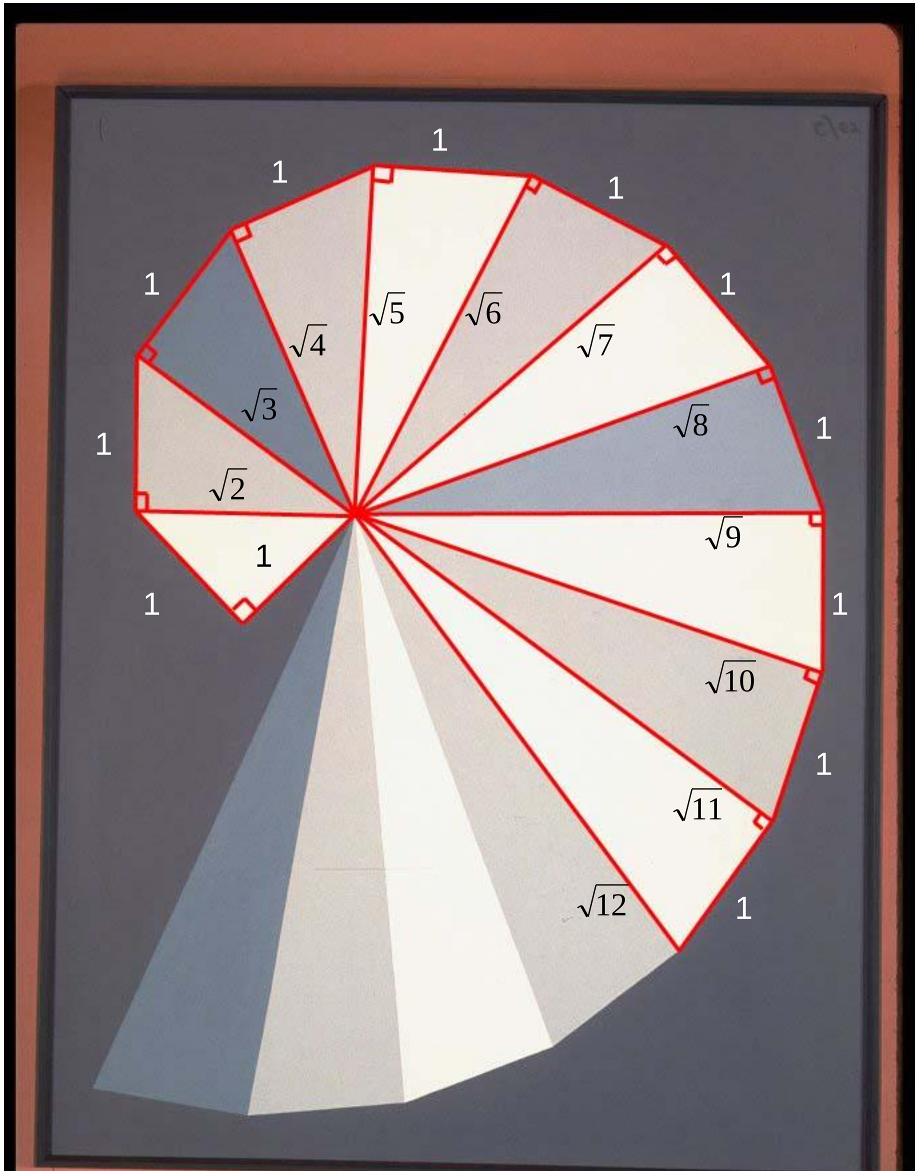

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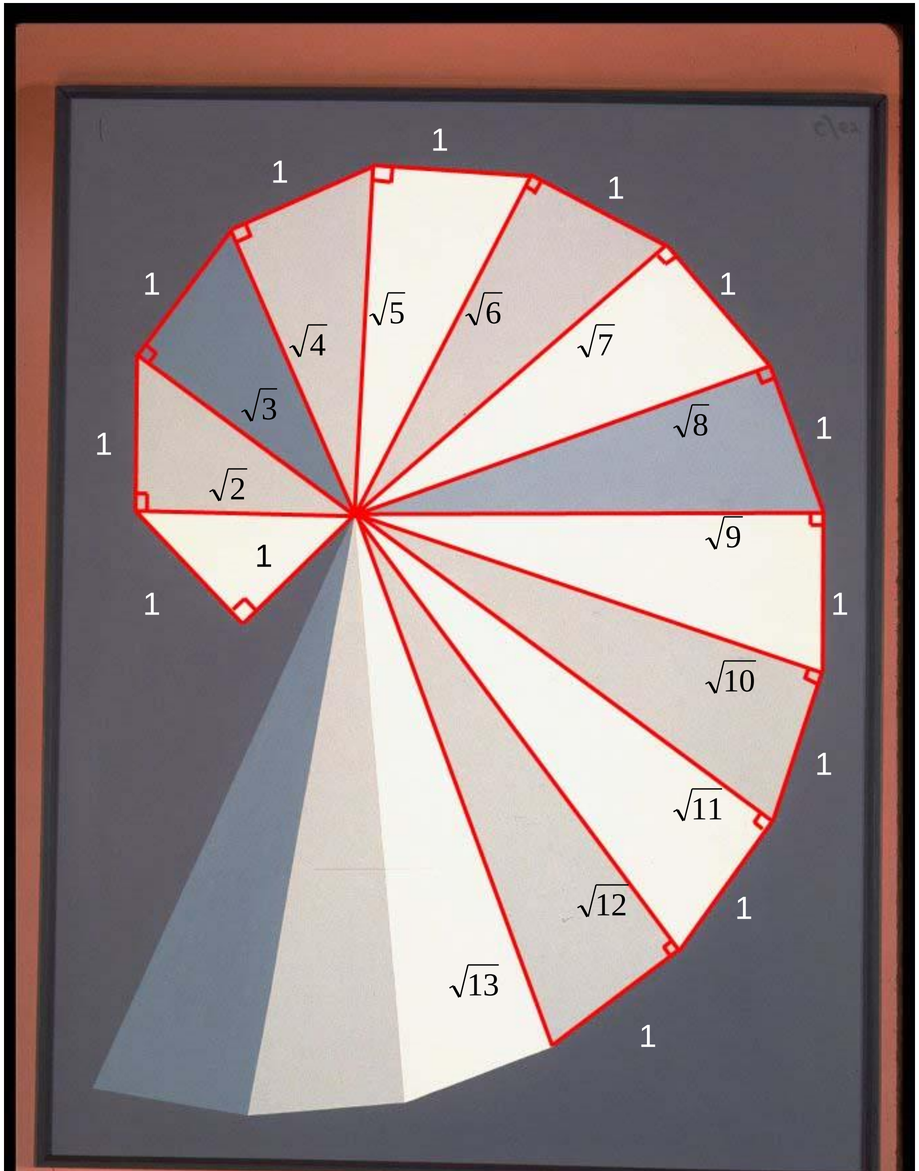

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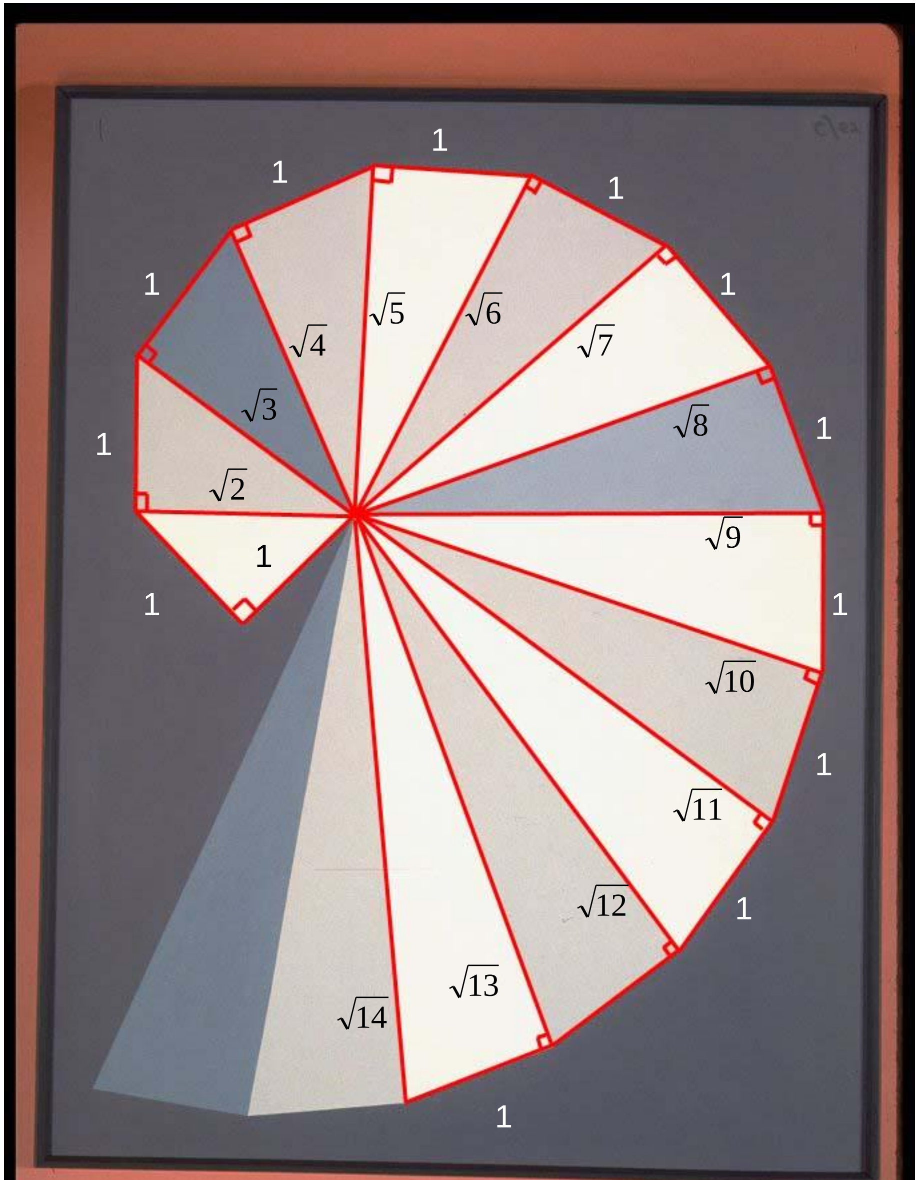

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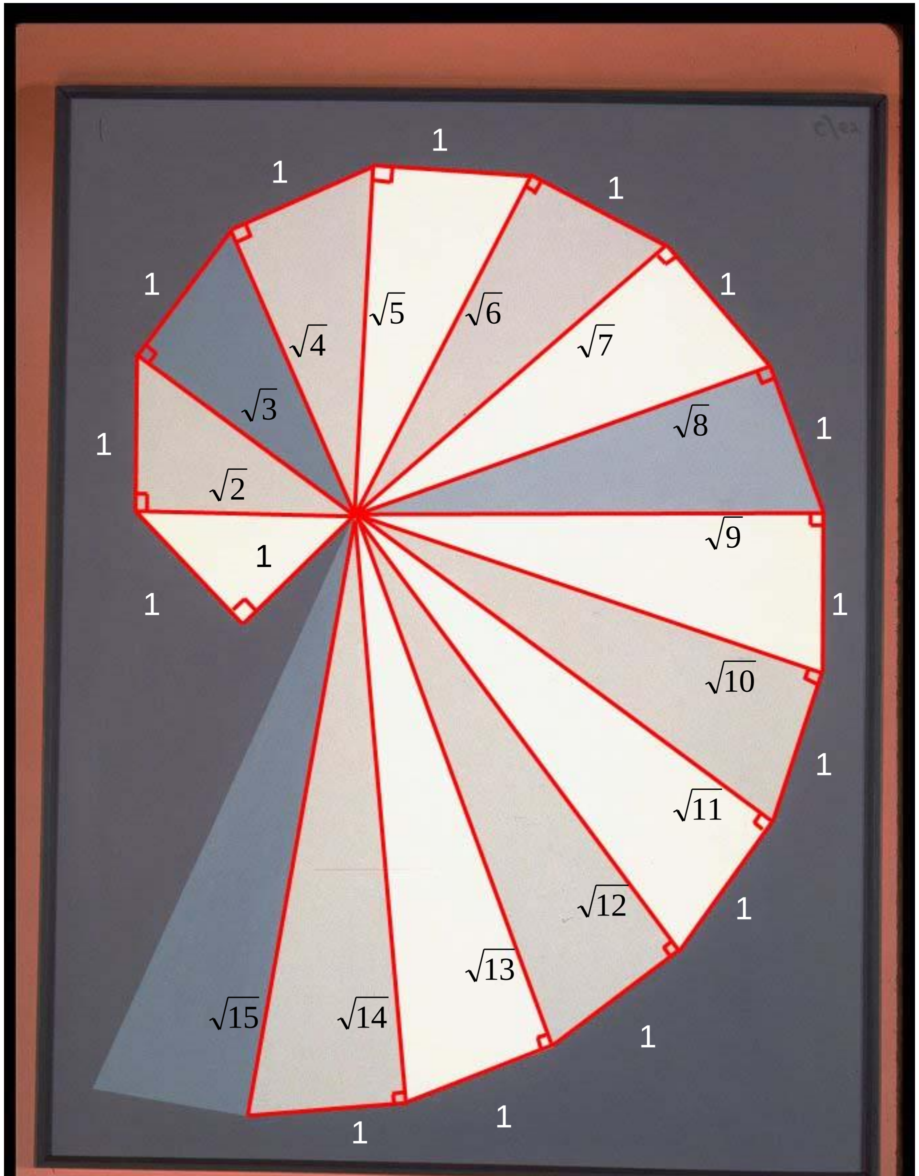

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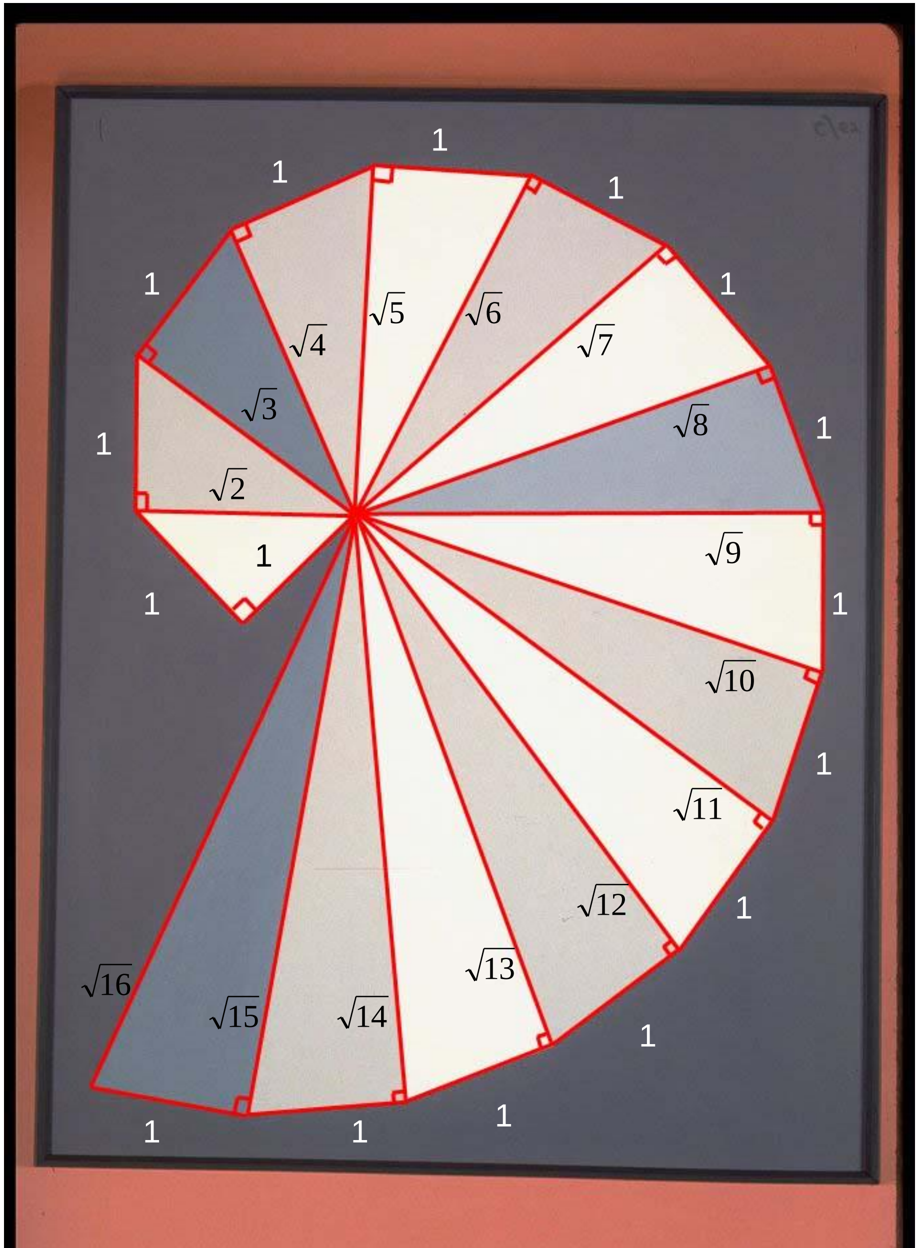


R. Calinger, *A Contextual History of Mathematics.*
Prentice Hall, 1999, p. 100.

Measurement of the Earth (Eratosthenes)

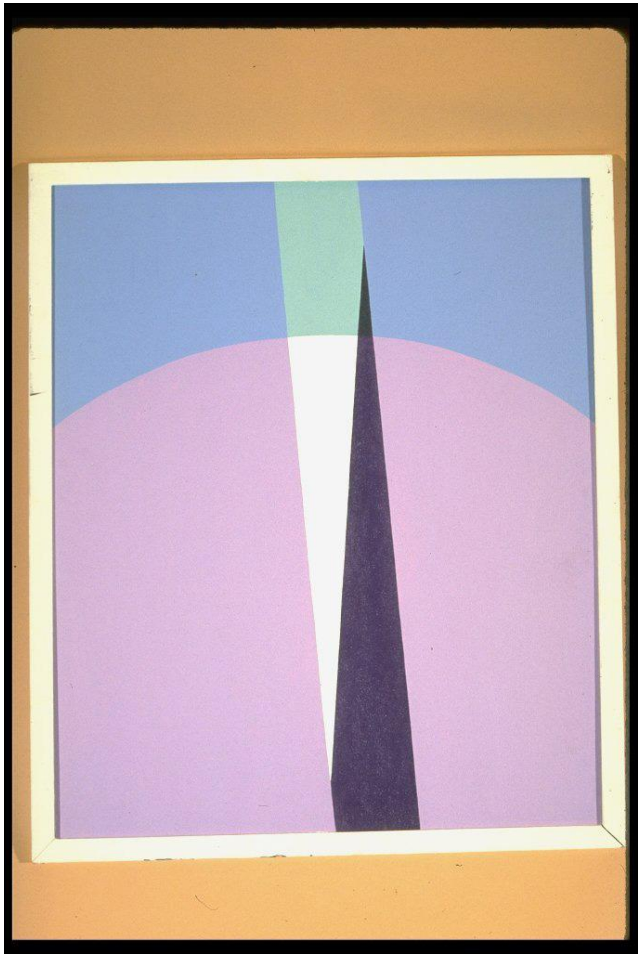
He observed that, while the sun cast no shadow from an upright gnomen in Syene at noon on the summer solstice, the shadow cast at the same at Alexandria… indicated an inclination of the sun's ray with the vertical to be 1/50 of the full circle, that is 7 degrees and 12 minutes. Hence

$$\frac{Circumference}{360} = \frac{5000\, stades}{\frac{360}{50}}$$

And therefore C=250,000 stades about 25,000 miles.

Calinger, p. 173-174.

Measurement of the Earth



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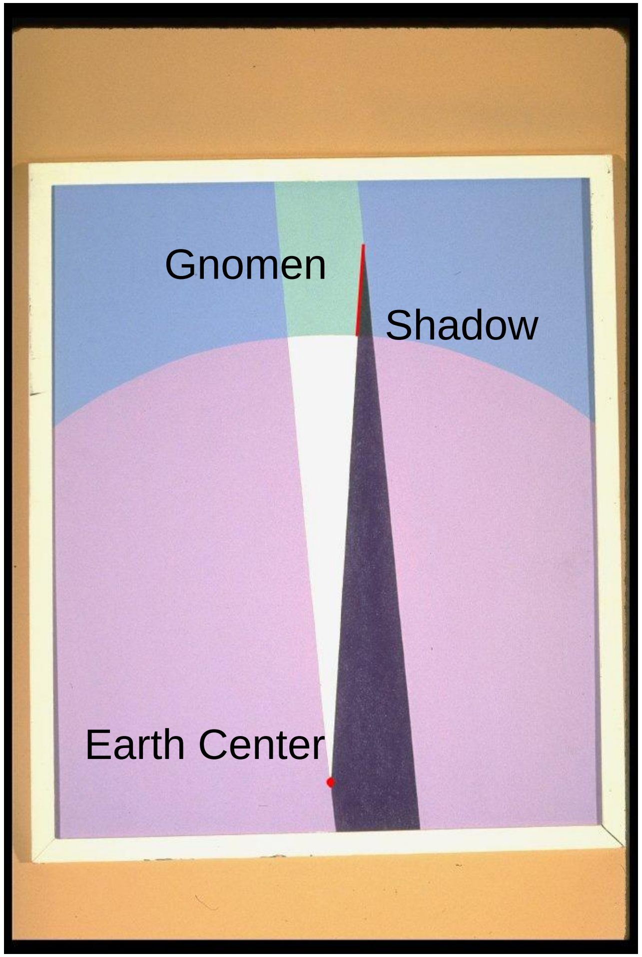

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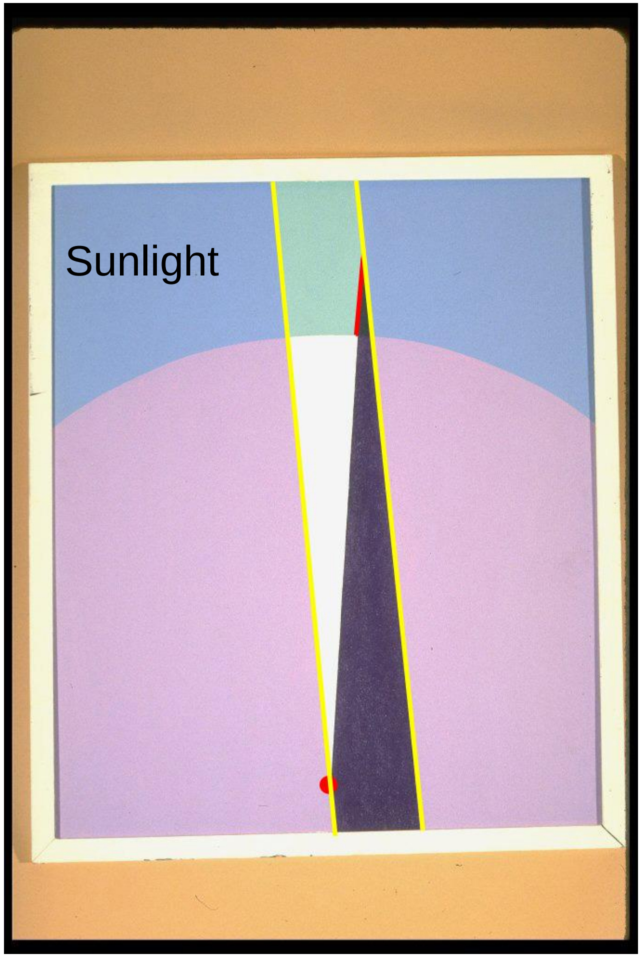

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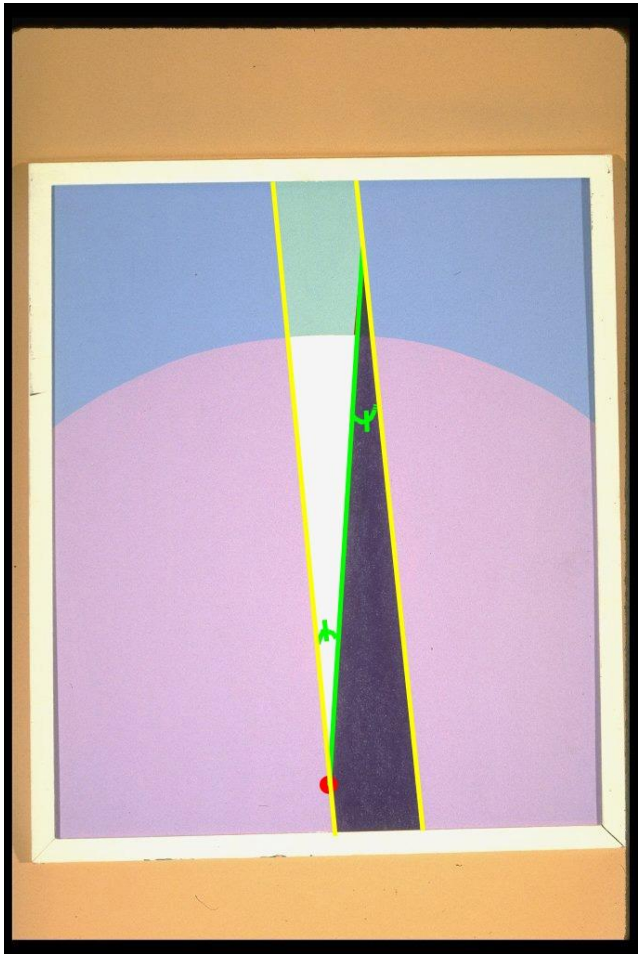


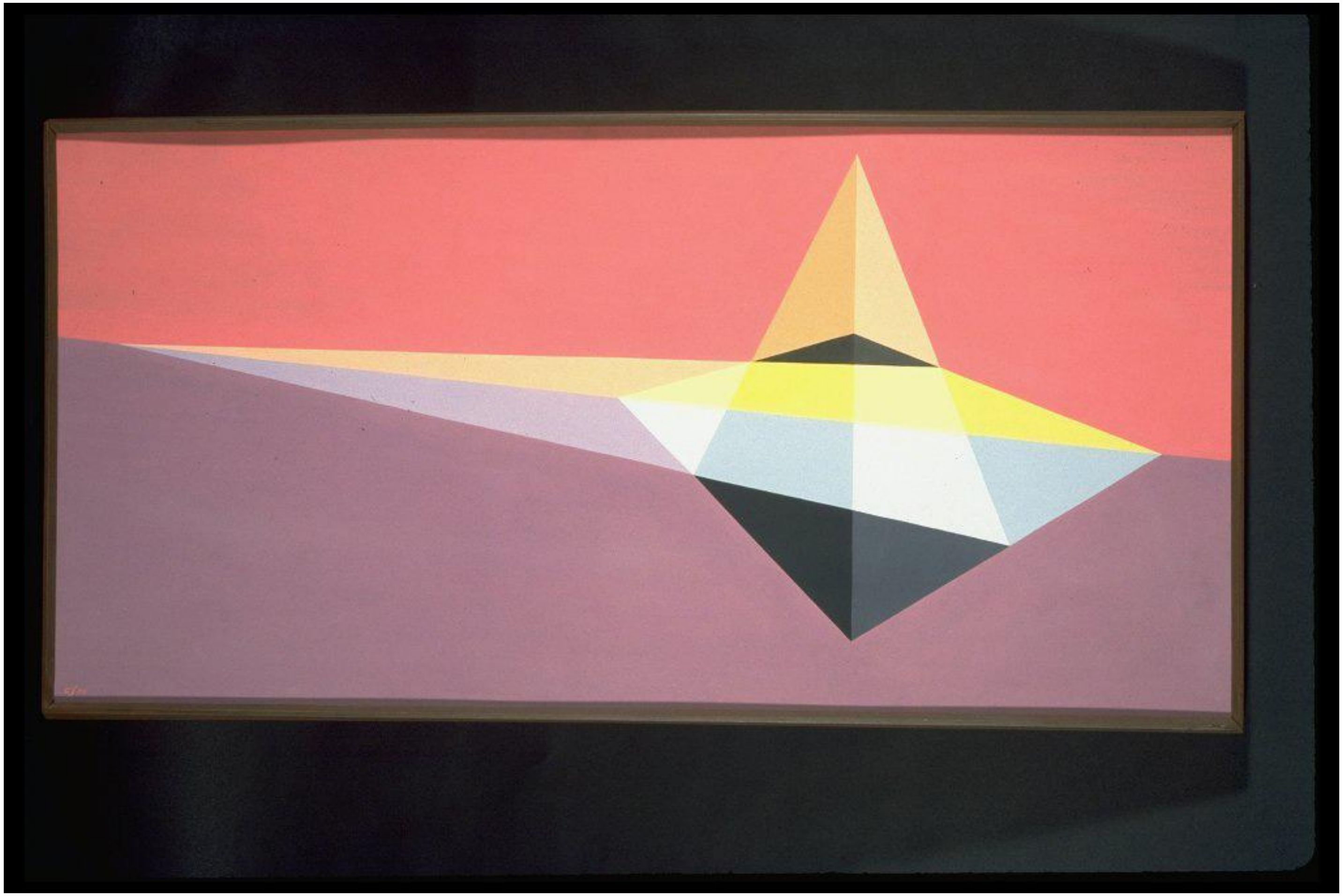
Parallel lines cut by a transversal.

Aligned Triangles

Desargues Theorem

If the corresponding vertices of two triangles ABC and XYZ lie on concurrent lines, the corresponding sides, if they intersect, meet in collinear points.

Aligned Triangles



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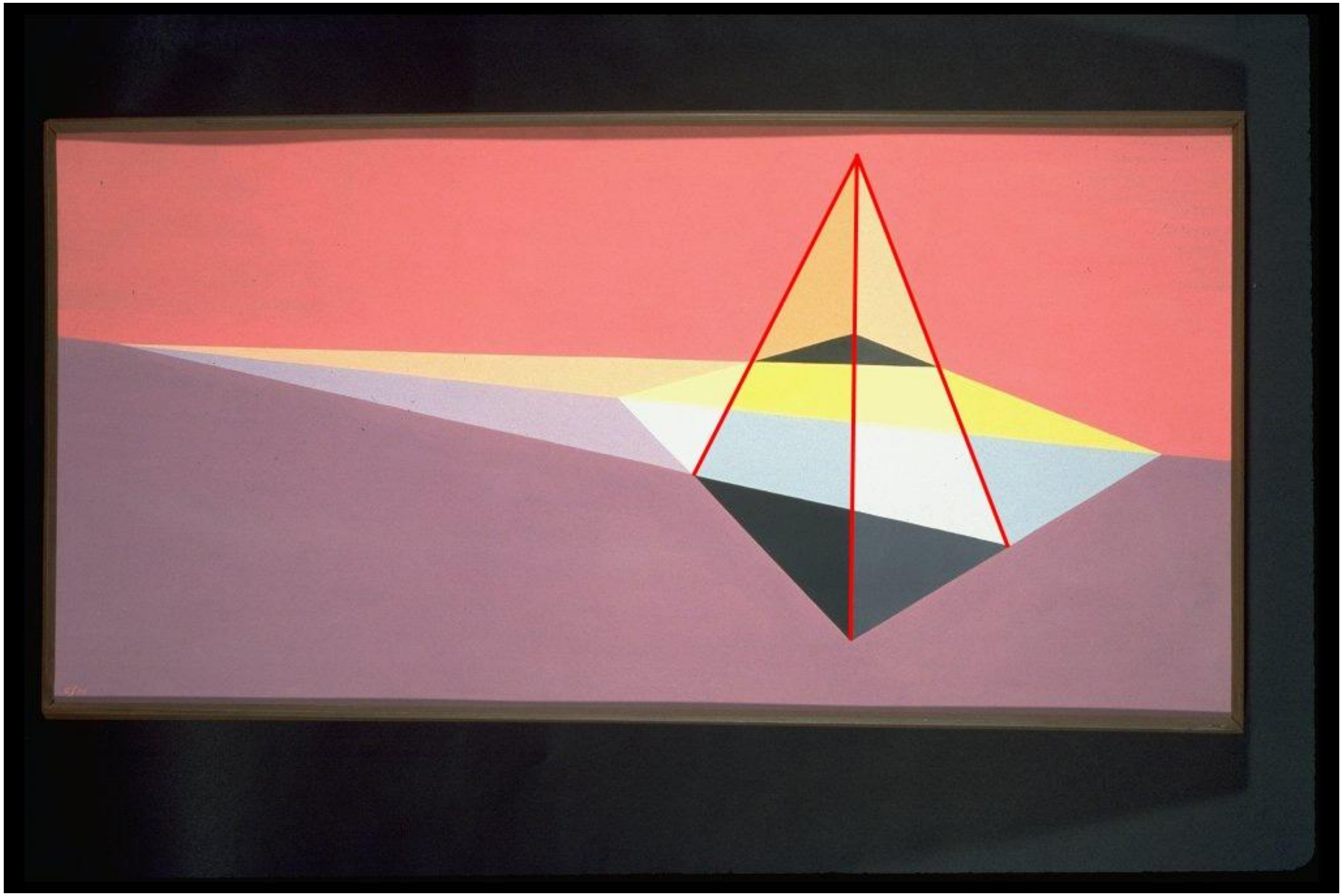

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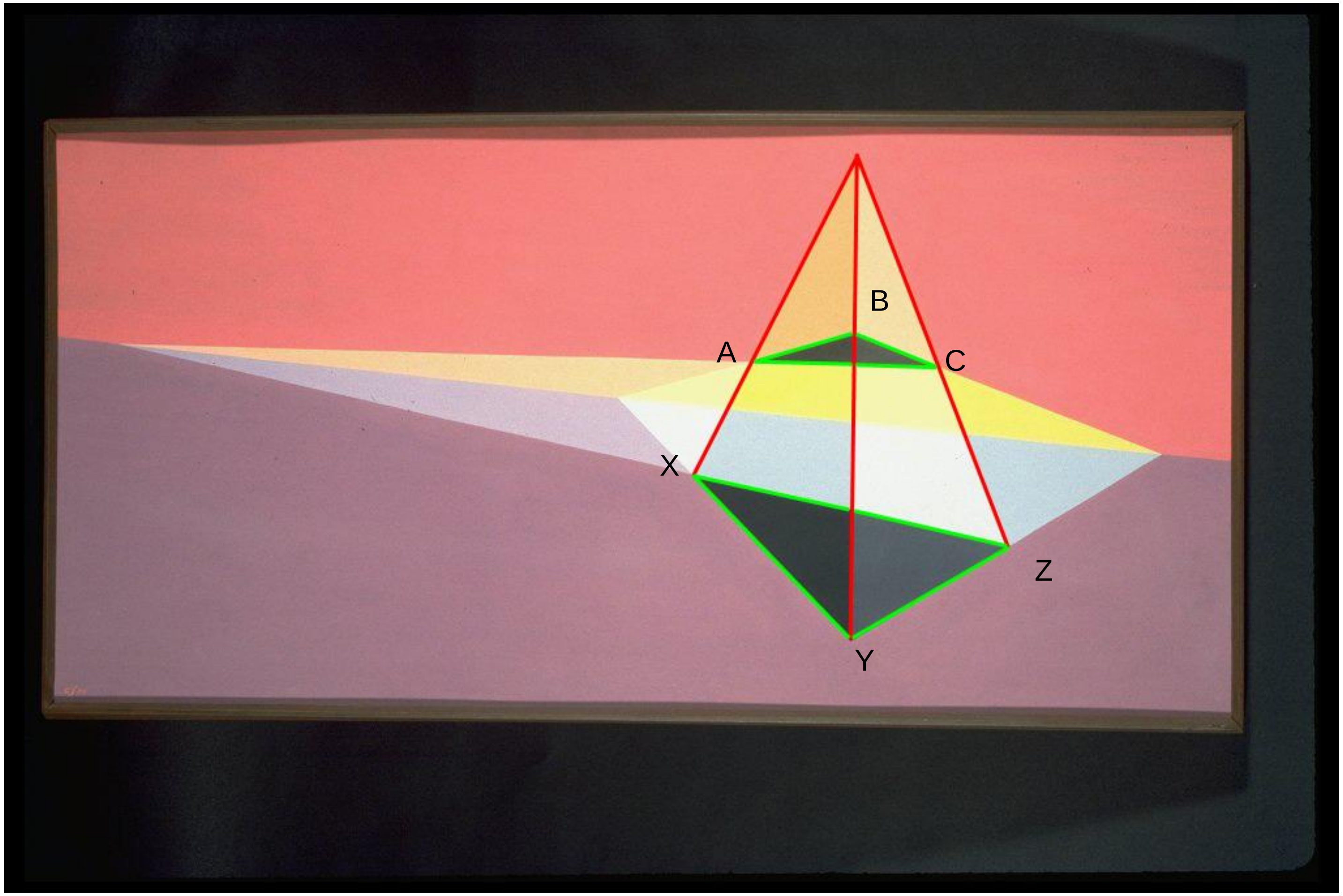

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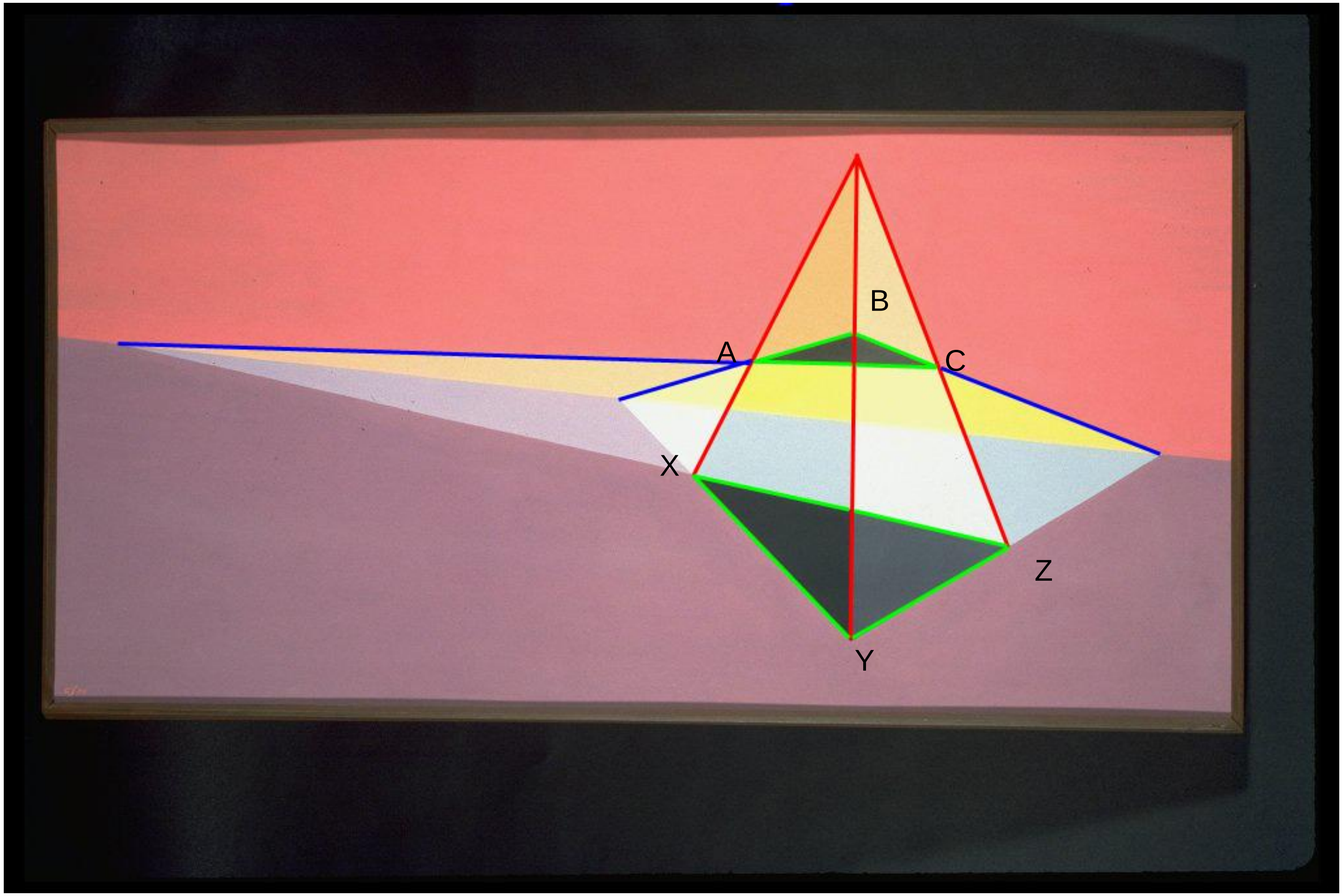
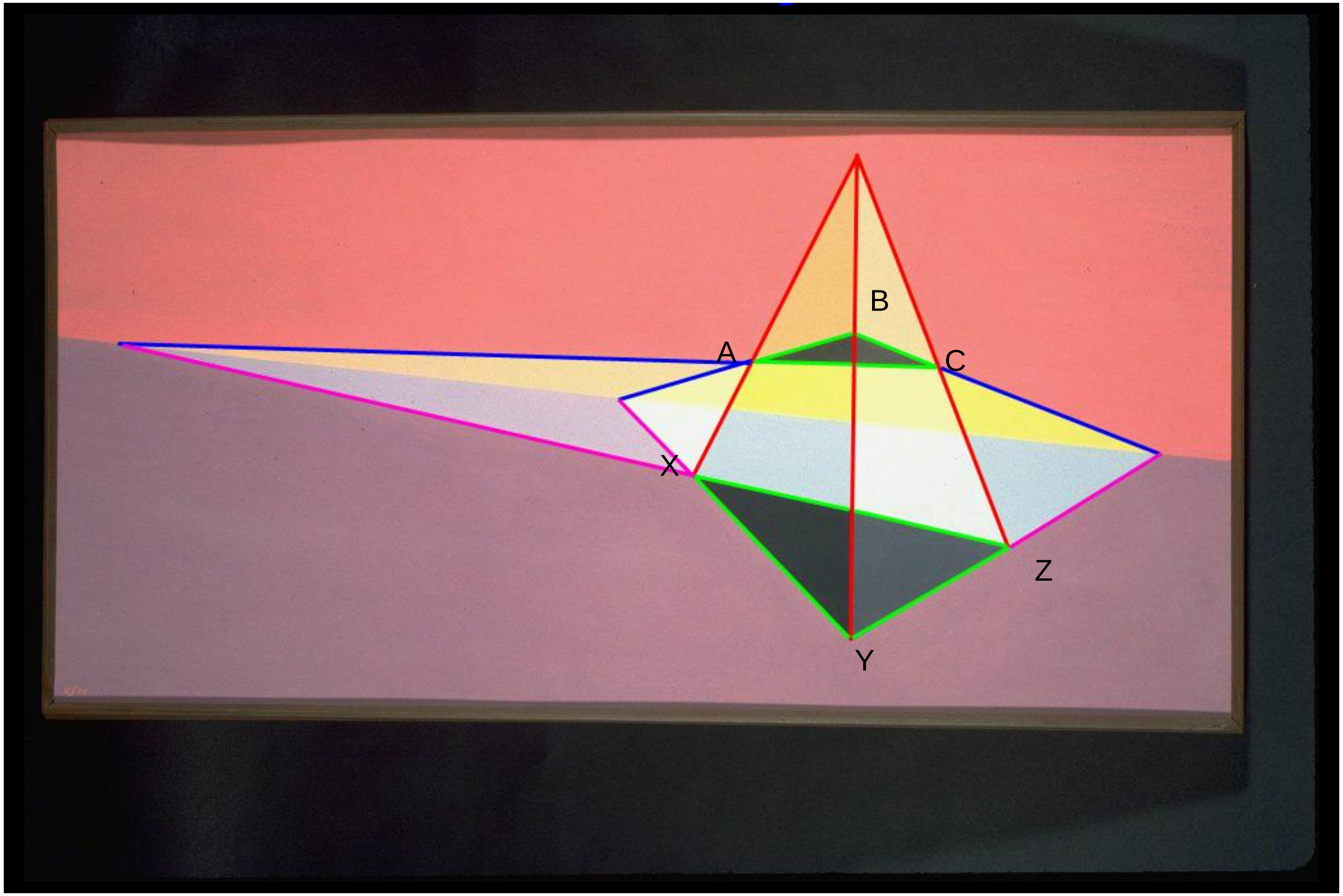


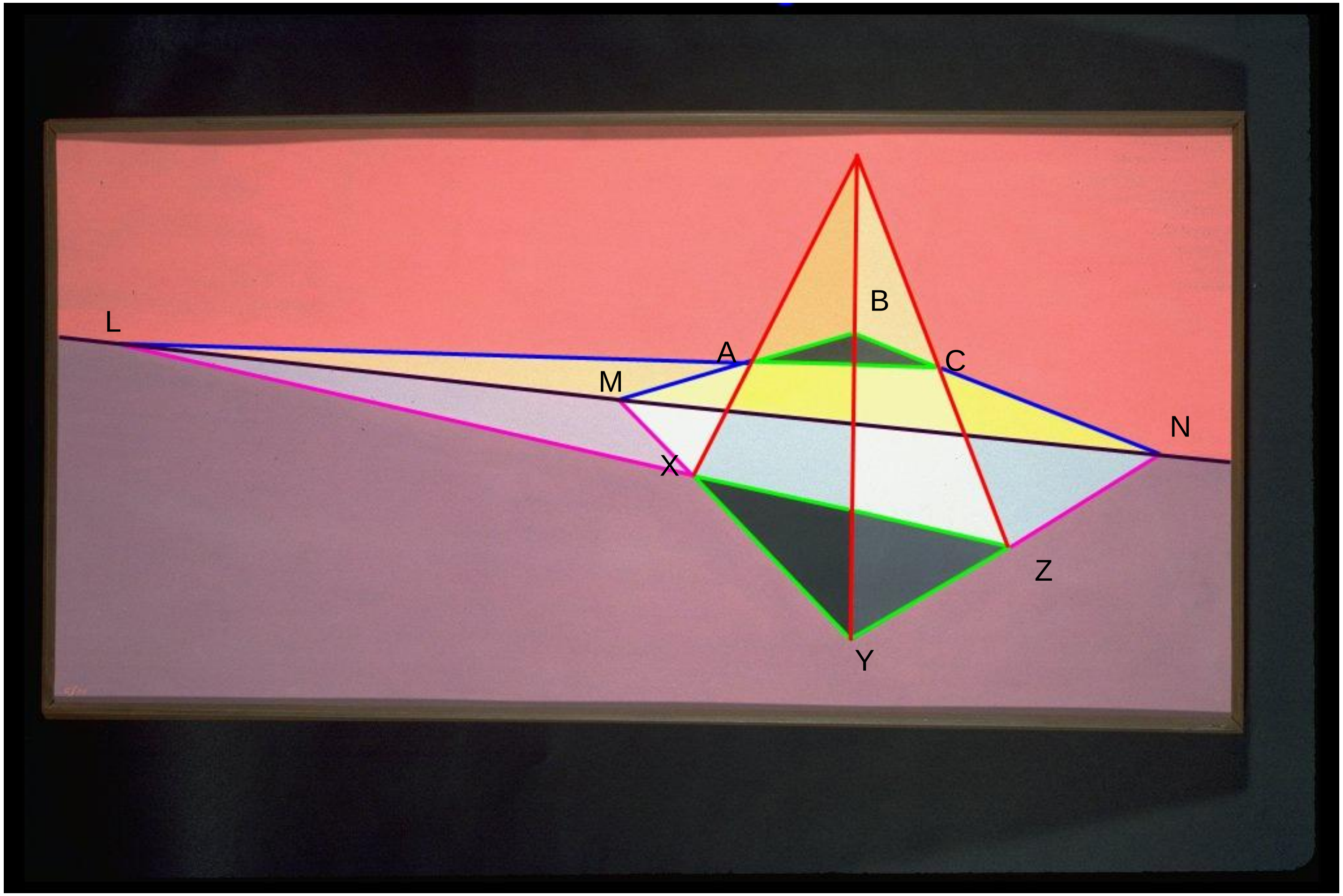
Extend the sides of triangle ABC.

Aligned Triangles



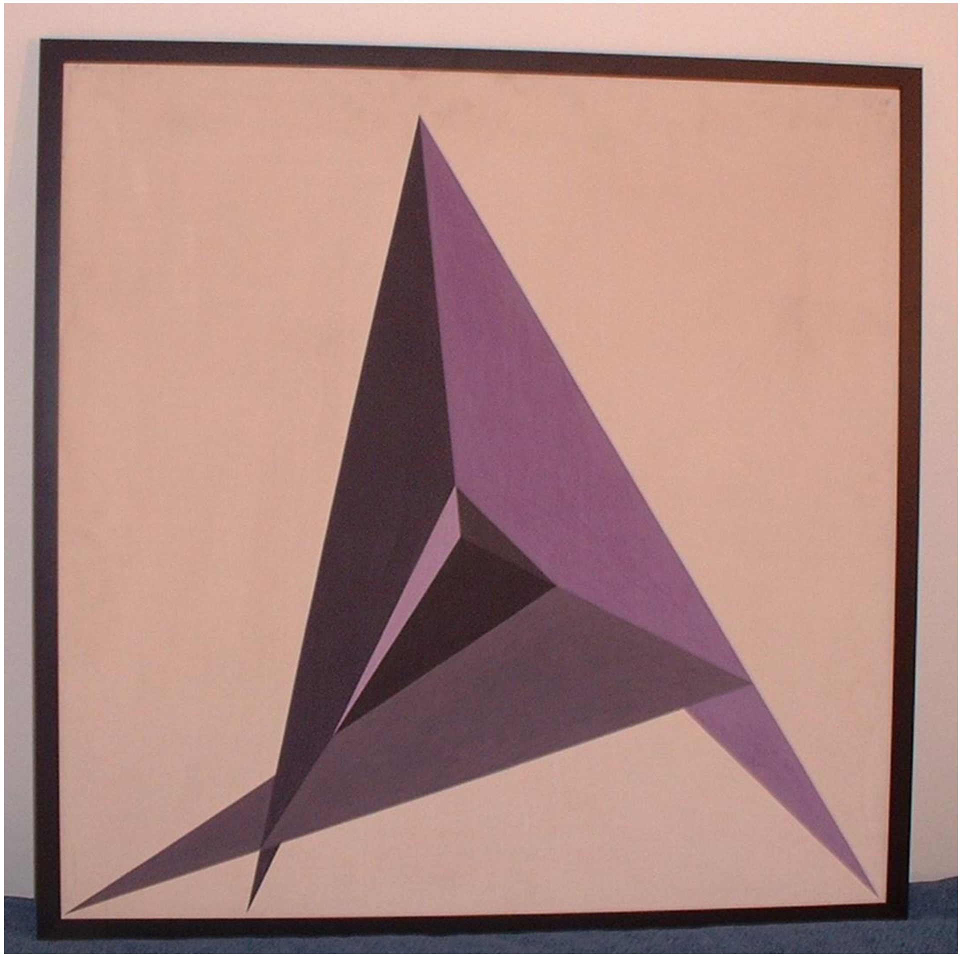
Extend the sides of triangle XYZ.

Aligned Triangles



L, M, N are collinear.

Desargues Theorem 2



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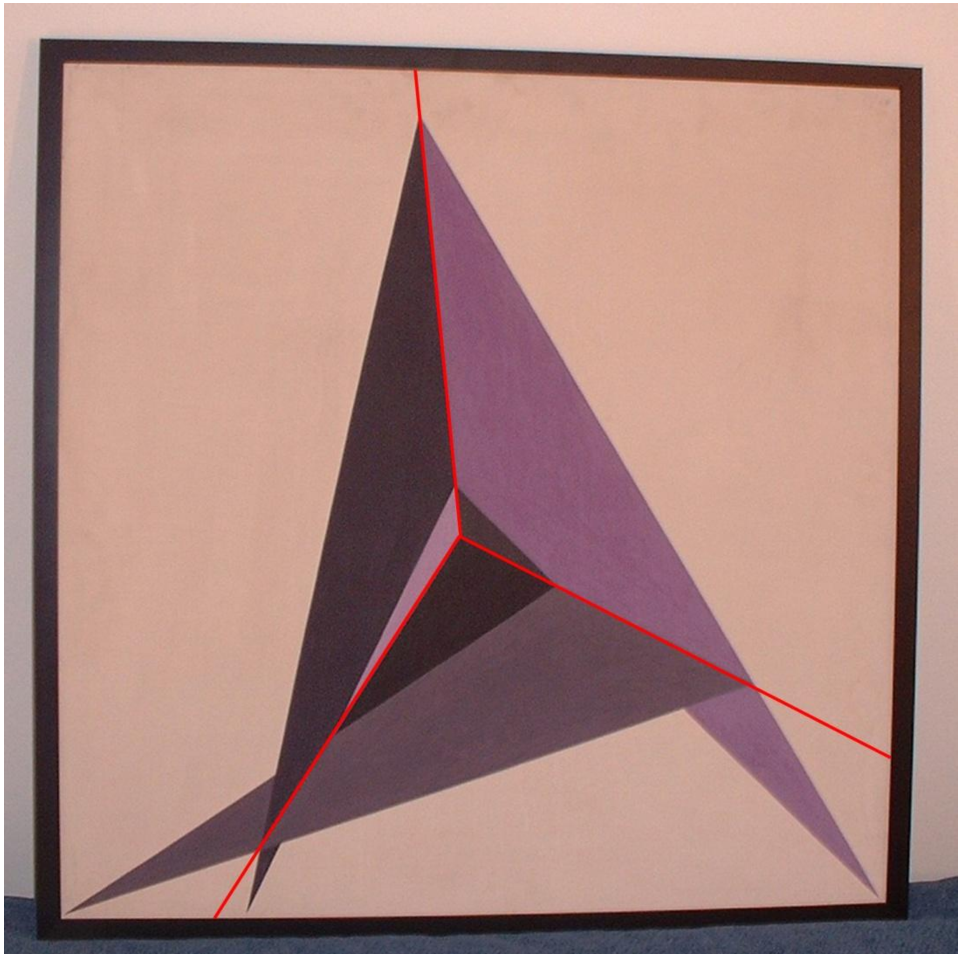

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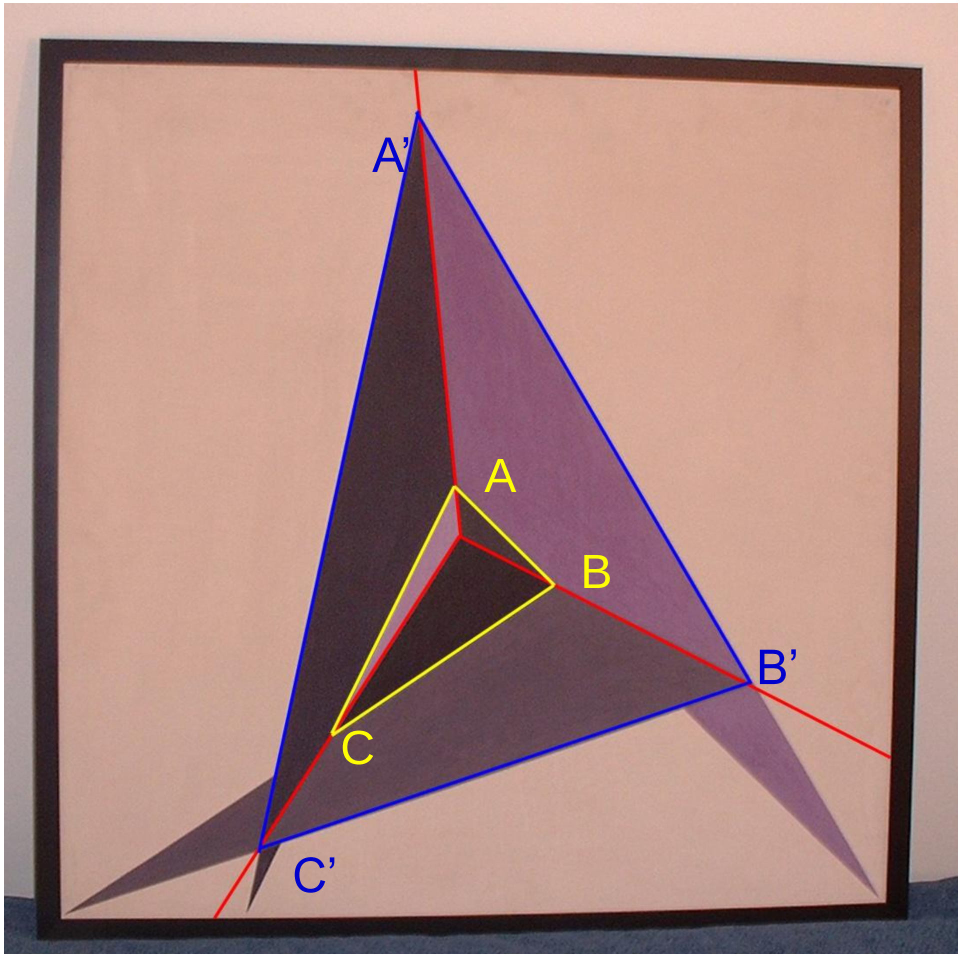
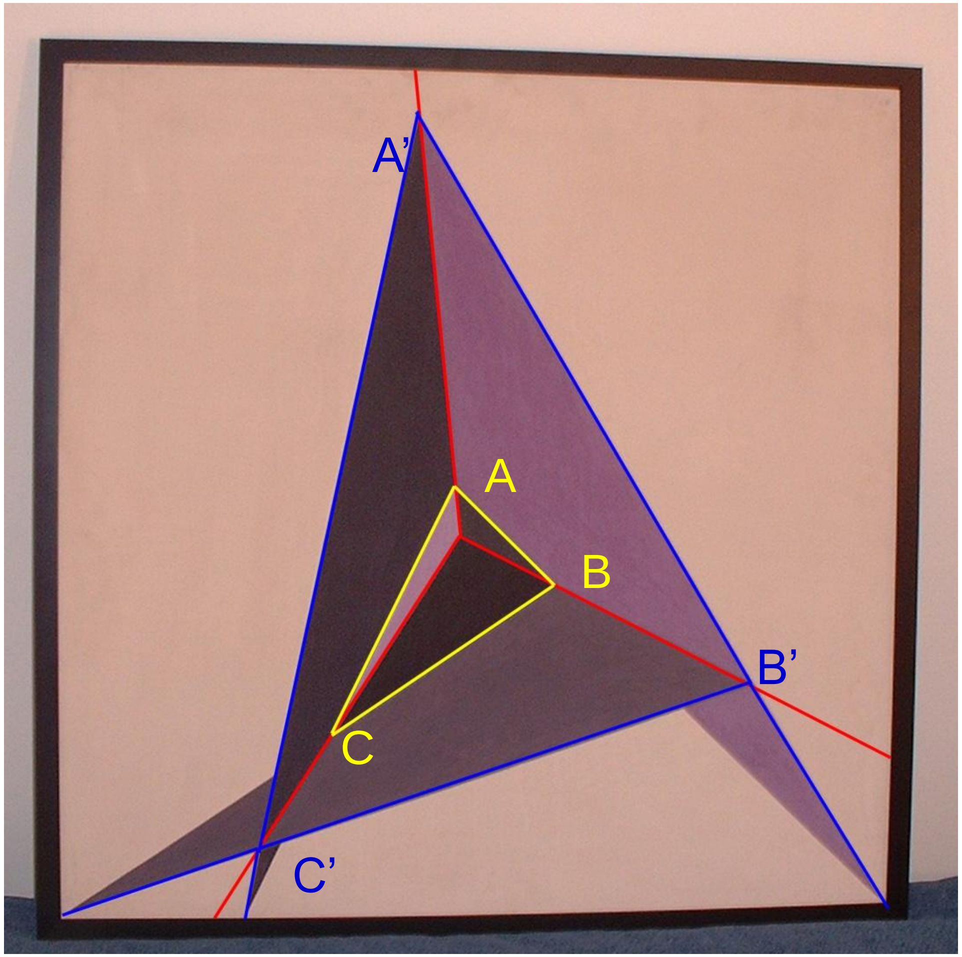


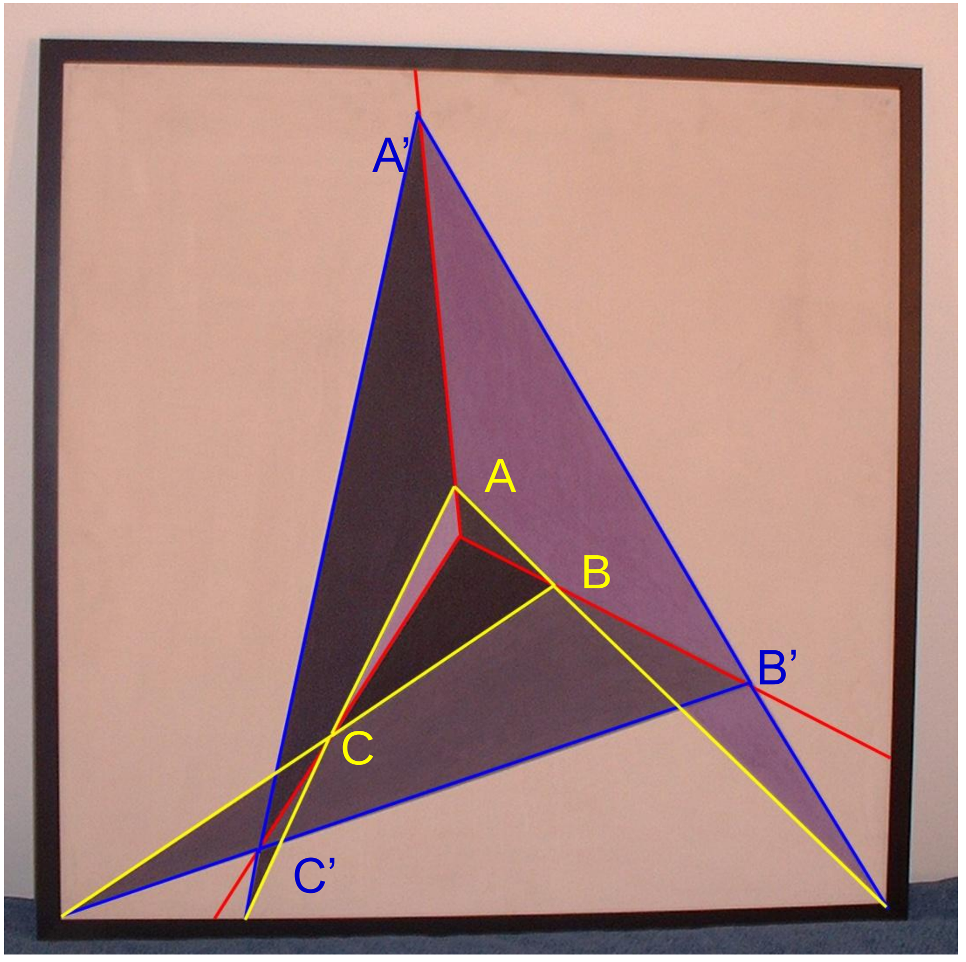
Desargues Theorem 2

Extend the sides of triangle A'B'C'



Desargues Theorem 2

Extend the sides of triangle ABC



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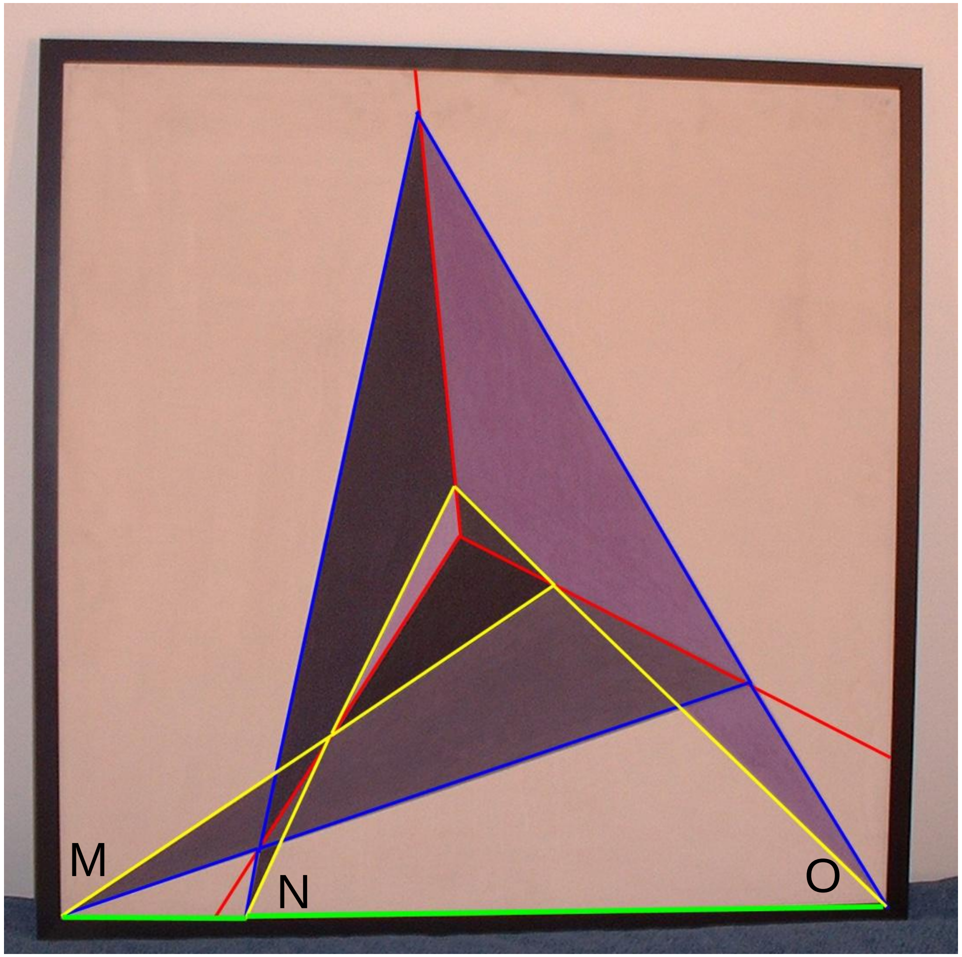


M, N, and O are collinear points.

Classical Greek Problems

- Problems date back to the time of the Pythagoreans, c. 530 B.C.E.
- Construction done with a straight edge and a compass.
- Resolution of theorems occurred in the nineteenth or twentieth century, with modern application of algebra and number theory.

Classical Greek Problems

1. Trisection of an arbitrary angle
2. Quadrature of a circle
3. Duplication of a cube
4. Quadrature of a lune
5. Construction of a regular polygon

Quadrature of the Circle

$$x^2 = \pi \cdot R^2$$

Let R = 1

$$x^2 = \pi$$

$$x = \sqrt{\pi}$$

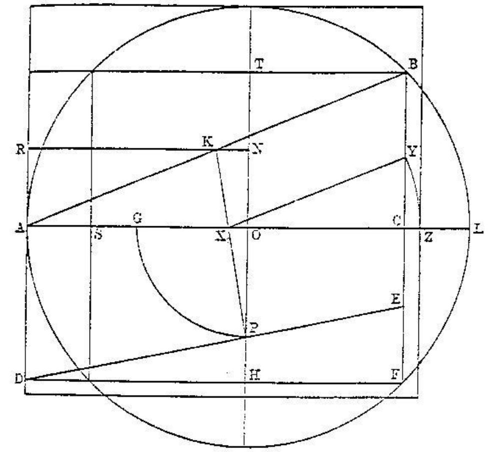
$\sqrt{\pi}$ **is Transcendental**

A Geometrical Look at Pi

The Mathematical Gazette in 1970 published "A Geometrical Look at Pi."

Johnson was very proud of this construction and used it in at least three paintings.

3250. A geometrical look at $\sqrt{\pi}$



$$SC = \sqrt{2}(1{\cdot}414214) = TH$$

$$AC = 1 + \frac{\sqrt{2}}{2}\,(1{\cdot}707107)$$

$$BC = \frac{\sqrt{2}}{2}\,(0{\cdot}707107) = TO = OC$$

$$AB = \sqrt{(2 + \sqrt{2})}(1{\cdot}847759)$$

$$KN = AO(1) - \frac{AC}{2} = \frac{2 - \sqrt{2}}{4}\,(0{\cdot}146447)$$

$$NP = \frac{1}{2} + \frac{\sqrt{2}}{4}\,(0{\cdot}853553)$$

$$\left(\frac{KN}{NP} = 0{\cdot}171573\right)$$

$$OP(0{\cdot}5)\,.\,\frac{KN}{NP} = XO(0{\cdot}085787)$$

$$AO(1) - XO = AX(0{\cdot}914213)$$

$$OC + XO = XC(0{\cdot}792894)$$

$$XY \parallel AB\left(\frac{AB}{AC} = 1{\cdot}082392\right)$$

$$XC\,.\,\frac{AB}{AC} = XY(0{\cdot}858222) = XZ$$

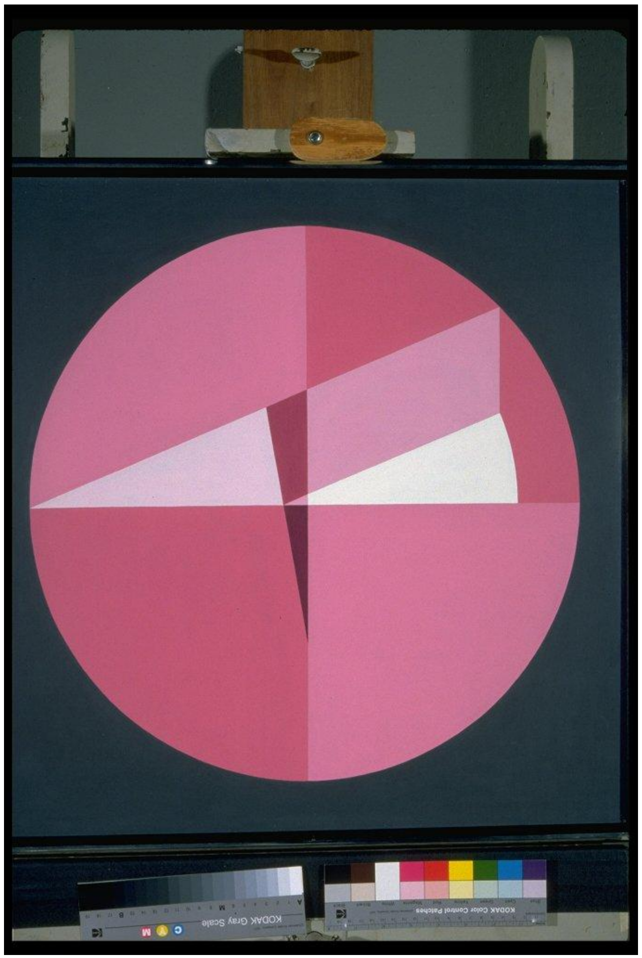
$$AX + XZ = AZ(1{\cdot}772435)$$

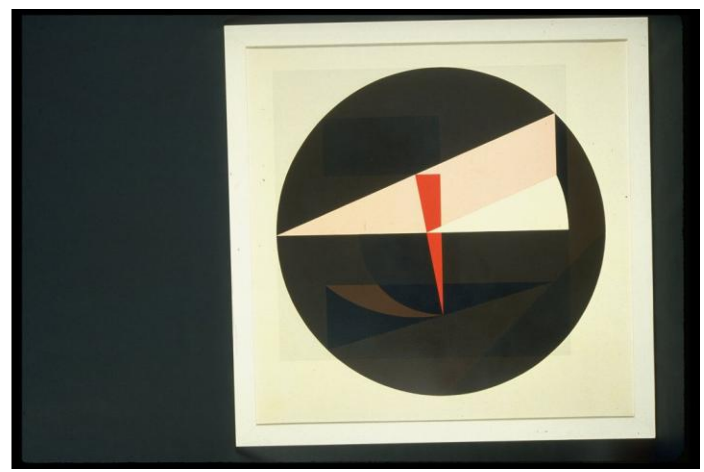
$$AZ = \sqrt{\pi} - 0{\cdot}00001$$

74 Rowayton Avenue,
Rowayton, Conn.

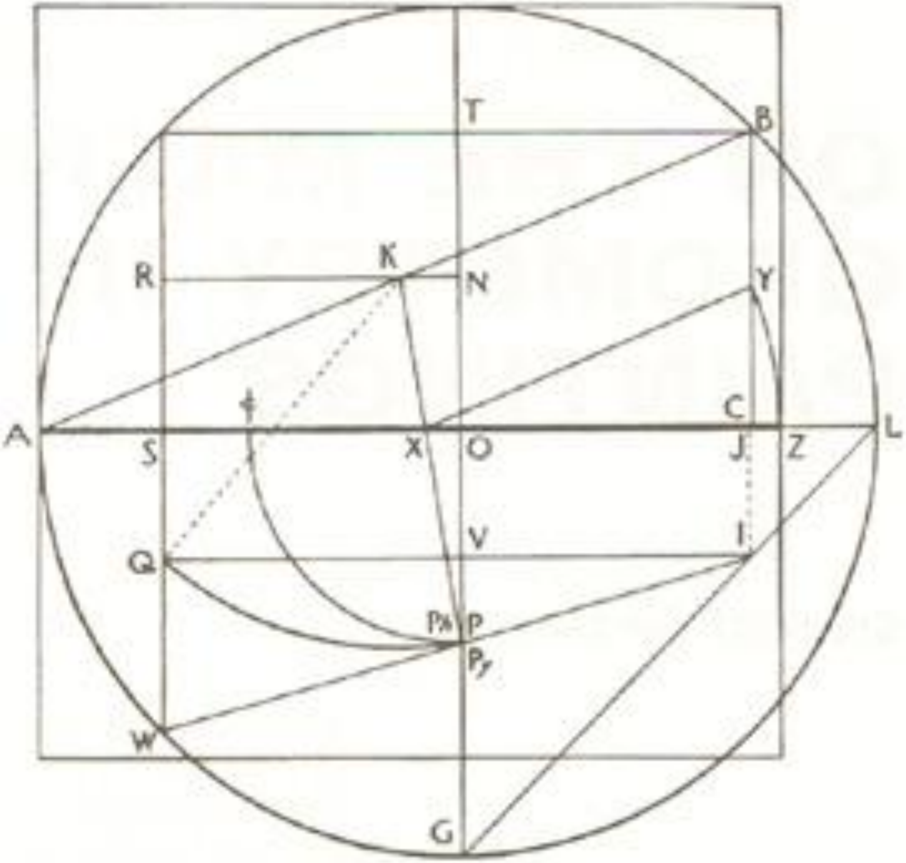
CROCKETT JOHNSON

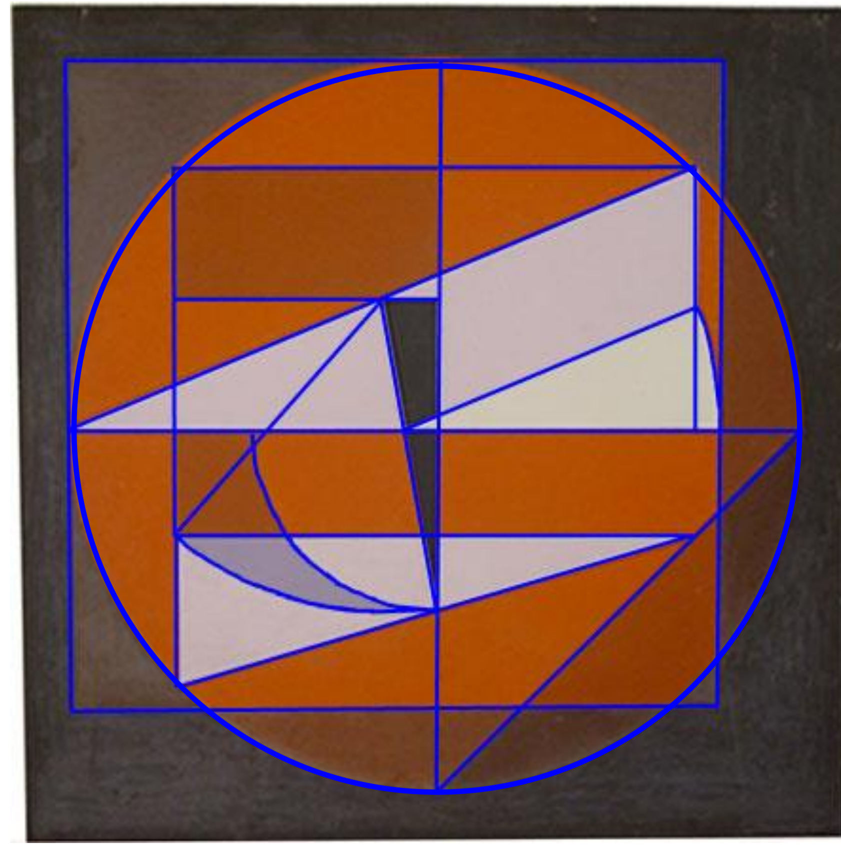
Square Root of Pi -.00001





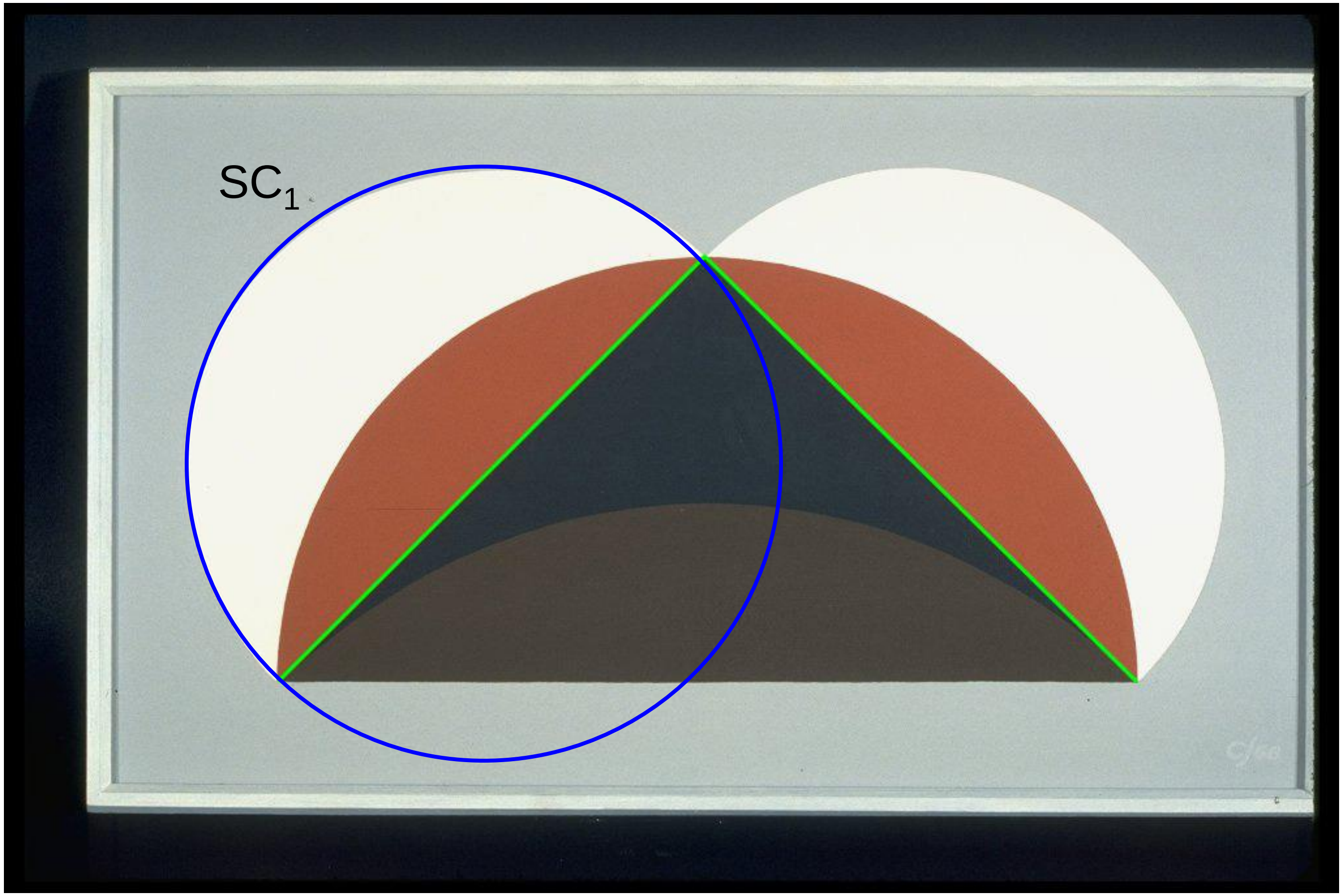
Square Root of Pi -.00001





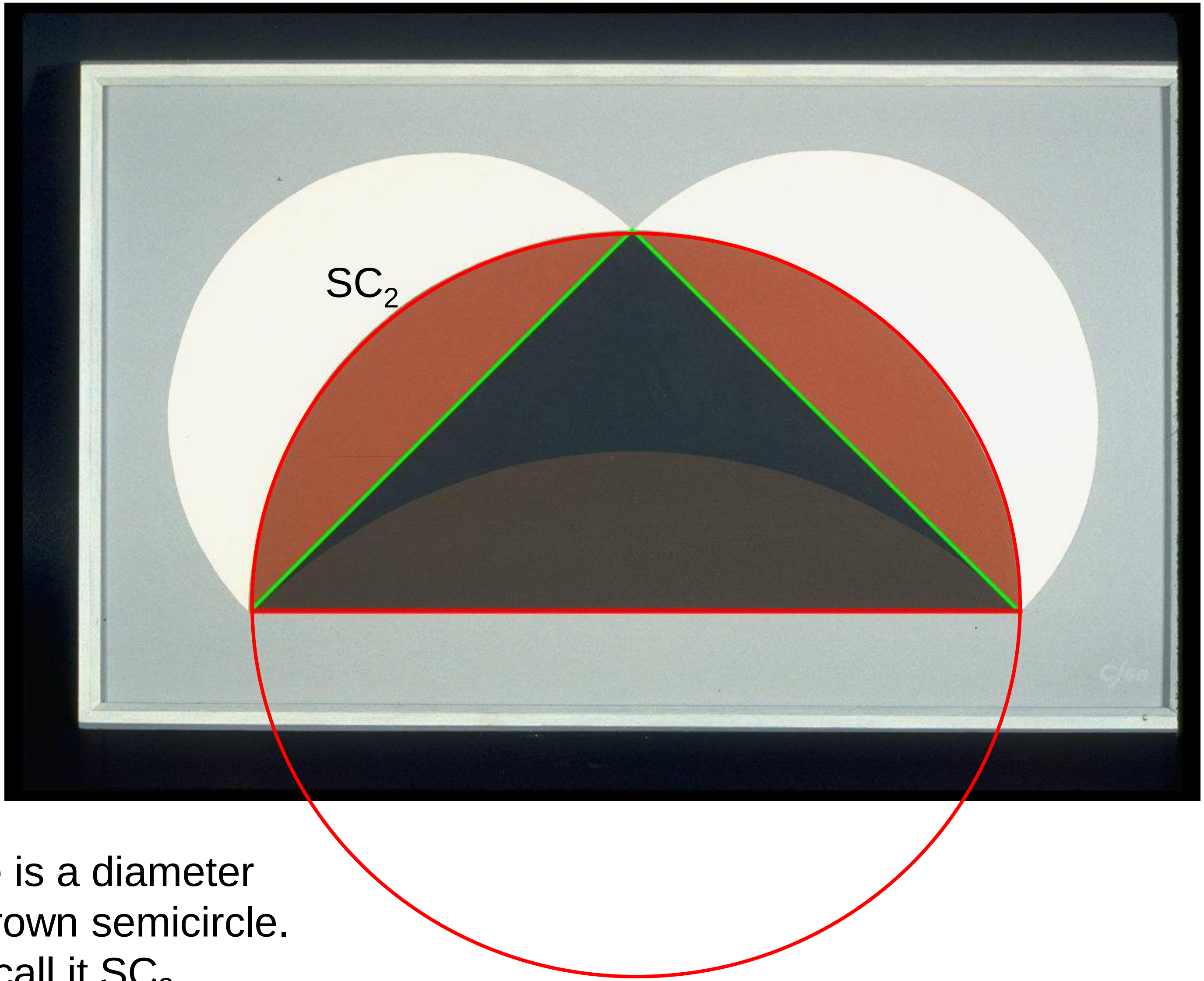
Quadrature of a Lune

Quadrature of a Lune



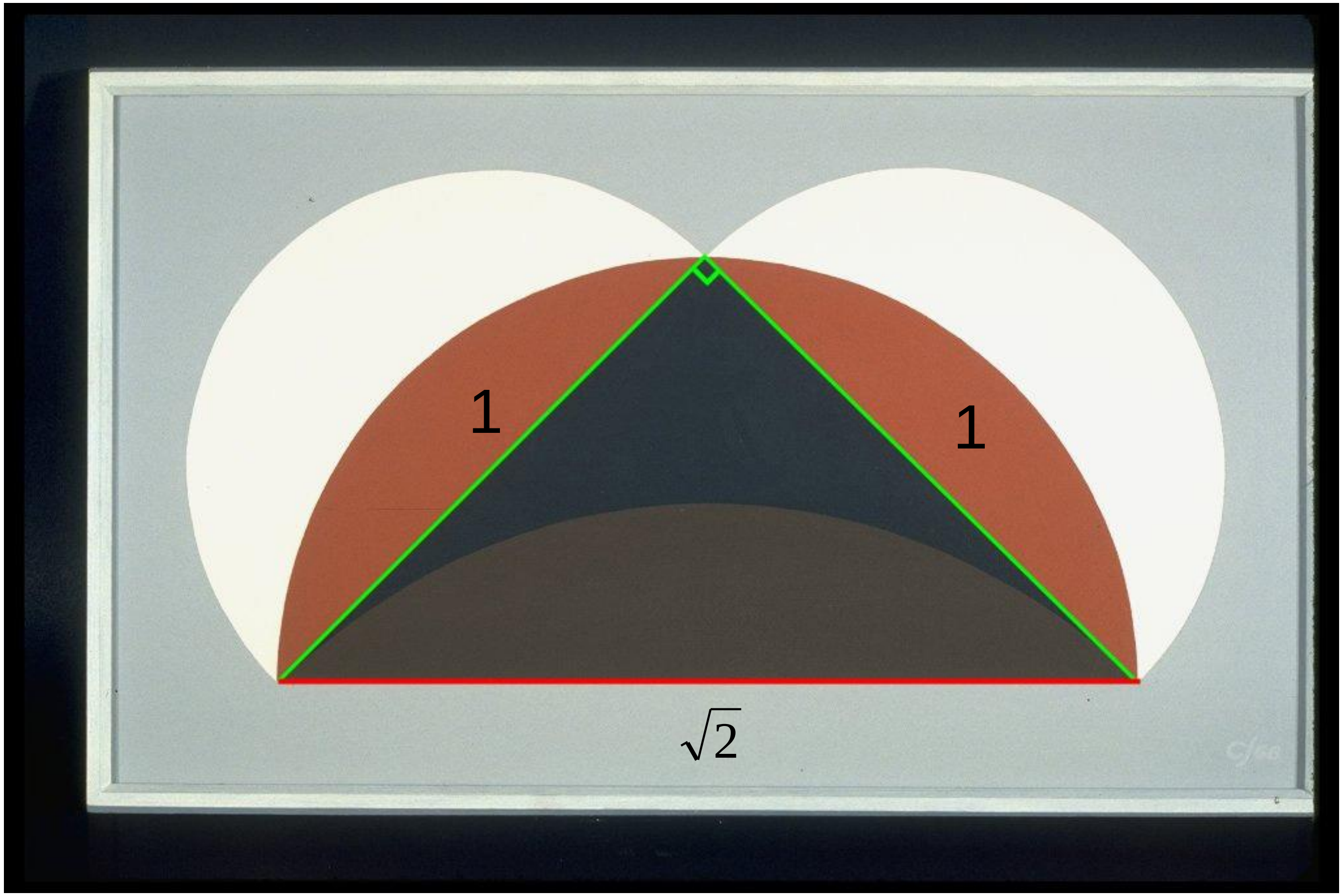
Green lines represent diameters to the semicircles.
We will call it SC_1.

Quadrature of a Lune



Red line is a diameter
to the brown semicircle.
We will call it SC_2.

Quadrature of a Lune



Some Proof:

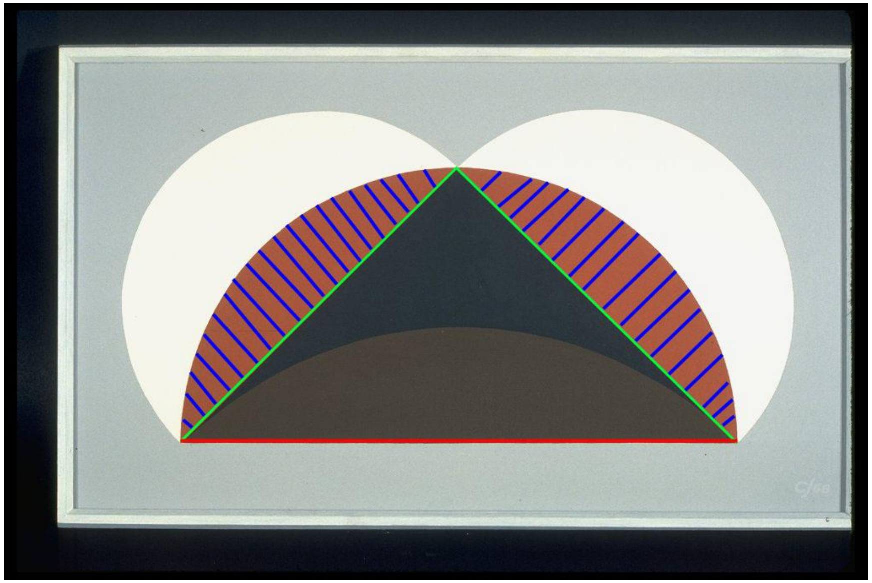
$$\frac{Area\ SC_1}{{D_1}^2} = \frac{Area\ SC_2}{{D_2}^2}$$

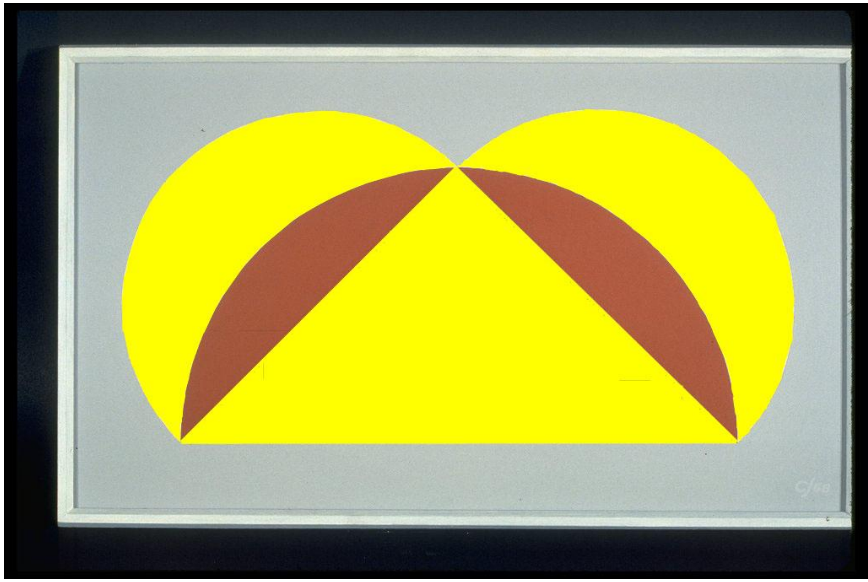
$$\Rightarrow \frac{Area\ SC_1}{1} = \frac{Area\ SC_2}{2}$$

$$\Rightarrow 2Area\ SC_1 = Area\ SC_2$$

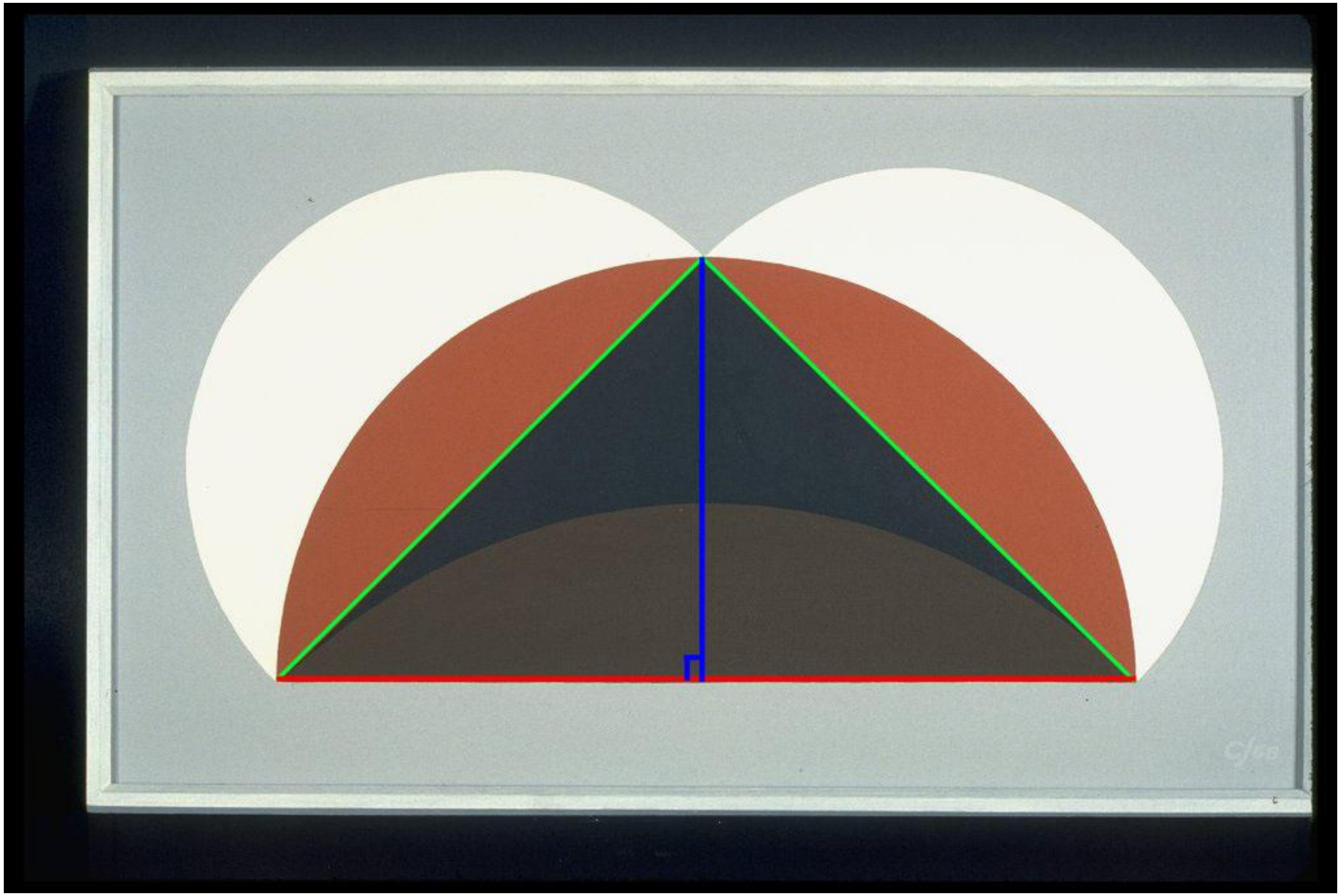
Therefore, the sum of the areas of the two smaller semicircles is equal in area to the larger semicircle.

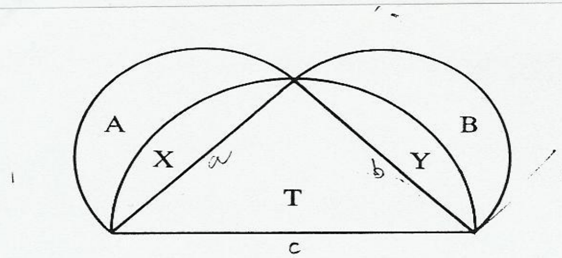
Quadrature of a Lune





Quadrature of a Lune

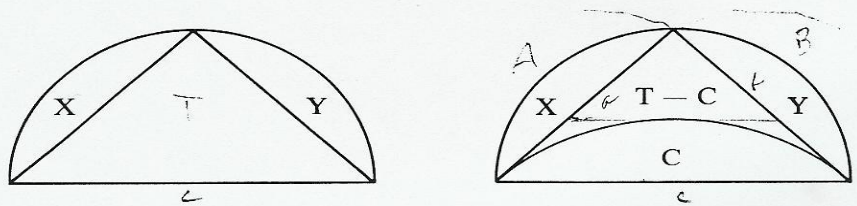




$$
\begin{aligned}
\text{semicircle on } a + \text{semicircle on } b &= \text{semicircle on } c \\
(A + X) + (B + Y) &= T + X + Y \\
A + B + (X + Y) &= T + (X + Y) \\
A + B &= T
\end{aligned}
$$

The last step is a matter of "squaring" a crescent, that is, constructing a square of equal area.

We start again with an isosceles right triangle inscribed in a semicircle. A circular segment, C, similar to segments X and Y, is constructed on the hypotenuse.



The entire semicircle is composed of two small circular segments, X and Y; a similar large segment, C; and $T - C$, the part of triangle "T" which lies above the large segment.

$$\text{semicircle on } c = X + Y + C + (T - C)$$

Each circular segment (X, Y, or C) is proportional to the square of its base (a, b, or c). And since $a^2 + b^2 = c^2$, then

$$\text{segment } X + \text{segment } Y = \text{segment } C$$

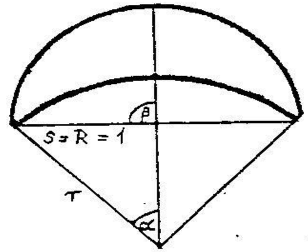
If we add $T - C$ to both sides of this last equation, we find that the crescent $X + Y + (T - C)$ equals triangle T.

$X + Y = C$

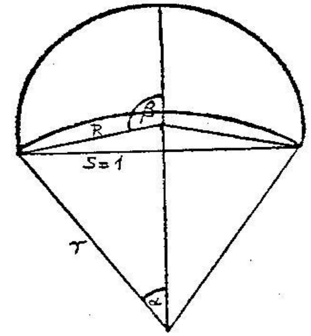
$X + Y + (T - C) = C + (T - c) = T$

The Five Squarable Circular Lunes

(m:n = 2:1, = 3:1, = 3:2, = 5:1, = 5:3)



2:1



3:1

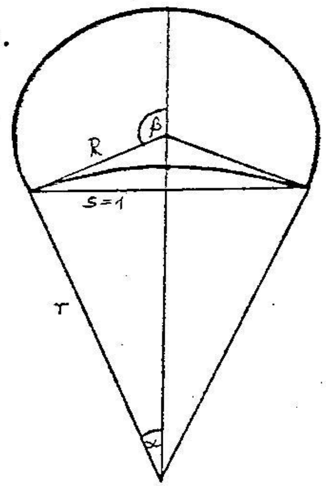
Common chord 2s (=2),

$r \cdot \sin\alpha = R \cdot \sin\beta = s = 1$.

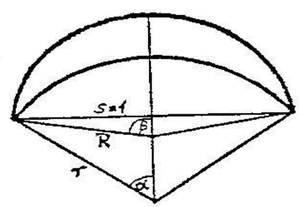
Circulars sectors of equal area:

$r^2\alpha = R^2\beta$; $\Rightarrow$

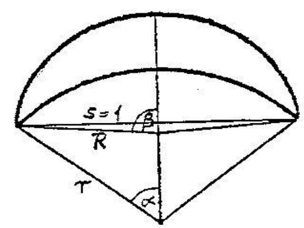
$\frac{R^2}{r^2} = \frac{\alpha}{\beta} = \frac{\sin^2\alpha}{\sin^2\beta} = \frac{m}{n}$.



5:1



3:2



5:3

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For more information

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